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ENDOGENOUS INFORMATION: THE ROLE OF SEQUENTIAL TRADE AND FINANCIAL PARTICIPATION

Autores: Sebastián Cea-Echenique, Carlos Hervés-Beloso y Juan Pablo Torres-Martínez

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ENDOGENOUS INFORMATION: THE ROLE OF SEQUENTIAL TRADE AND FINANCIAL PARTICIPATION

SEBASTIÁN CEA-ECHENIQUE, CARLOS HERVÉS-BELOSO, AND JUAN PABLO TORRES-MARTÍNEZ

ABSTRACT. We develop a general equilibrium model with differential information and incomplete financial participation. Agents endogenously update information from commodity prices and financial contracts. Without require financial survival assumptions, we prove equilibrium existence in a model where the heterogeneity of preferences across states of nature may depends on available information.

KEYWORDS. Incomplete Financial Participation - Endogenous Differential Information
JEL CLASSIFICATION: D52 - D53 - D82

1. INTRODUCTION

There is a large literature on competitive equilibrium with differential information, a framework introduced by Radner (1968) where agents may not distinguish the state of nature that was reached after the realization of the uncertainty. Allowing for incomplete financial markets and sequential trade, Faias and Moreno-García (2010) propose an extension of the original model of differential information. They analyze the compatibility between equilibrium prices and common ex-ante information in nominal asset markets, showing that equilibria with non-informative prices exist and that the degree of real indeterminacy of equilibrium decreases.

On the other hand, financial imperfections produced by a lack of information, credit risk, or emerging from regulatory considerations, induce financial participation constraints. With the aim to study these situations, the general equilibrium model of incomplete financial markets[1] was extended to scenarios where agents have personalized access to financial opportunities (see, for instance, the pioneering works of Siconolfi (1989) and Cass (1984, 2006)).[2] In this context, Seghir and

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[1]The theory of incomplete financial markets starts with Radner (1972), Drèze (1974) and Hart (1975), which extended Arrow and Debreu (1954) to allow for an incomplete set of financial promises (see for instance, Geanakoplos (1990) and Magill and Quinzii (2008) for surveys of major results in this literature).

[2]Equilibrium models with incomplete financial participation were also studied by Balasko, Cass, and Siconolfi (1990), Polemarchakis and Siconolfi (1997), Angeloni and Cornet (2006), Aouani and Cornet (2009), and Cornet and Ranjan (2011), requiring financial survival assumptions to equilibrium existence. Aouani and Cornet (2011)

Torres-Martínez (2011) propose a model with credit participation constraints where financial survival conditions are not required: assuming that individuals are impatient, they prove equilibrium existence even when agents do not have access to all credit contracts.

The objective of our work is to address the existence of equilibrium in a model with differential information and incomplete financial participation, where agents deduce new information on the realization of uncertainty from market signals. They obtain information from financial assets, through state contingent payments.[3] Also, since individuals perfect foresight prices, spot commodity prices reveal information after the realization of uncertainty. Our framework extends the model of Faias and Moreno-García (2010) by allowing for endogenous updating of agent's private information and enlightening equilibrium prices. We also extend Seghir and Torres-Martínez (2011) to include investment constraints. Thus, individuals obtain information only from financial instruments that they know and can trade.[4]

As in the classical static model of differential information, in our context there is a compatibility between consumption bundles and private information. In contrast to the Radner (1968) model, in our framework agents demand consumption plans that are compatible with their final private information, which could have been improved by updating endogenously their initial information.[5]

Furthermore, we argue that the heterogeneity of agent's tastes across states of nature can be reduced as a consequence of the lack of information. That is, we contemplate the situation in which individual's objective functions depend on the information that spot commodity prices reveal. Therefore, although preferences are exogenous, the subjective prior belief of agents or the relative importance given to contingent consumption could depend on the final information available. Also, we consider the case in which individual preferences may be affected by relative commodity prices,

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and Cornet and Gopalan (2012) impose spanning conditions over portfolio sets. Price dependent constraints were addressed by Cass, Siconolfi, and Villanaci (2001) and Carosi, Gori, and Villanacci (2009).

[3]The presence of assets in our model allow us to avoid some of the important restrictions we face with the model by Radner (1968). For instance, the example of no-trade caused by the lack of information in Correia-da-Silva and Hervés-Beloso (2009) does not apply if the obvious assets were to be considered.

[4]Physical and financial markets providing new information to incomplete informed traders have been previously studied in other contexts, by Radner (1979), Polemarchakis and Siconolfi (1993), Rahi (1995) and Cornet and De Boisdeffre (2002).

[5]Correia-da-Silva and Hervés-Beloso (2009) show that compatibility between consumption and information can be endogenized also in another framework, allowing for uncertain delivery of commodities and provided that individuals have prudent expectations on market deliveries.

as in the classic work by Pollak (1977)[6] or Correia-da-Silva and Hervés-Beloso (2008) for a model of preferences for list of bundles.

The rest of the paper is organized as follows: in Section 2 we introduce the model and in Section 3 we discuss our main result and its assumptions. We conclude with some remarks in Section 4 and prove our main result in the Appendix.

2. Model

Consider a two period economy without uncertainty in the first period, $t = 0$, and where one state of nature of a finite set S is realized in the second period, $t = 1$. To reduce notations, let $S^* = \{0\} \cup S$ be the set of states of nature in the economy, identifying $s = 0$ as the only state of nature in the first period. There is a finite set L of commodities that may be traded at each period in spot markets. Let $p_s = (p_{s,l}; l \in L)$ be the vector of unitary commodity prices at state of nature $s \in S^*$ and $p = (p_s; s \in S^*)$ the set of commodity prices in the economy. Hereinafter, we fix a bundle $\zeta \in \mathbb{R}^L_{++}$ and normalize unitary prices in such form that $p_s \cdot \zeta = 1, \forall s \in S^*$. Thus, the set of commodity prices will be $\mathcal{P} := \{(p_s; s \in S^*) \in \mathbb{R}^{L\times S^*}_{+} : p_s \cdot \zeta = 1,\ \forall s \in S^*\}$.

There is a finite set J of real assets. Each asset $j \in J$ issued at $t = 0$, has a unitary price q_j, and makes promises contingent to the states of nature, $(R_{s,j}\,\zeta; s \in S) \in \mathbb{R}^{L\times S}_{+} \setminus \{0\}$. Let $q := (q_j; j \in J) \in \mathbb{R}^J_+$. We assume that there are no redundant assets. That is, that the family of vectors $\{(R_{s,j})_{s\in S}; j \in J\}$ is linearly independent.

There is a finite set of agents, denoted by I. Each agent $i \in I$ may have incomplete information about the realization of the uncertainty and has restricted financial participation. On one hand, each agent $i \in I$ only distinguishes the states of nature that are in different elements of a partition $\mathbb{P}^i$ of S, although all the relevant information about the states of nature is the pooling of individuals information, i.e., $\bigvee_{i\in I} \mathbb{P}^i = \{\{s\}; s \in S\}$.[7] Additionally, agent i has a $\mathbb{P}^i$-measurable endowment of commodities $w^i = (w^i_s; s \in S^*) \in \mathbb{R}^{L\times S^*}_{++}$ (i.e., endowments do not add additional information). On the other hand, individual i only knows and can trade assets in a subset $J^i \subseteq J$. We assume that asset returns are signals that may be used to update information. We suppose, without loss of generality, that for each $i \in I$, the partition $\mathbb{P}^i$ incorporates all the information generated by

[6] Even in economies with perfect and complete information, preferences may be affected by relative prices as a signal of quality, social status, or externalities. Under these contexts, equilibrium existence and properties of competitive equilibria was studied by Shafer and Sonnenschein (1975), Greenberg, Shitowitz, and Wieczorek (1979), Balder (2003), Balasko (2003), Cornet and Topuzu (2005) and Noguchi (2009), among others.

[7] Since preferences will endogenize the information compatibility requirement (see Assumption (A1) below), we do not need to assume that for any $s \in S$ there is $i \in I$ that distinguish it, i.e., $\{s\} \in \mathbb{P}^i$. This is a traditional assumption on static general equilibrium models with differential information, used to ensure that (under monotonicity of preferences) the equilibrium price of any contingent commodity contract is strictly positive.

the return of assets in J^i. That is, for any $j \in J^i$, the vector $(R_{s,j}; s \in S)$ is $\mathbb{P}^i$-measurable.[8] Consequently, the information obtained by the returns of financial assets is incorporated into the private information. Thus, as is assumed in Faias and Moreno-García (2010), the assets that an agent can trade do not reveal new information.

We assume, that for any $j \in J$, there is $i \in I$ such that $j \in J^i$. However, it is possible for some agents $i \in I$, $J \setminus J^i \neq \emptyset$. Therefore, as in Seghir and Torres-Martínez (2011), we do not impose any kind of financial survival assumption. Notice, that assets in $J \setminus J^i$ may contain information that agent i does not have.

In our model, information matters since, it may affect prior beliefs on the probability of occurrence of each state and because agents may have heterogeneous state dependent preferences. However, to take advantage of this variability of tastes, agents need to have the capacity to recognize all the states of nature. Thus, we assume that individual preferences depend on the endogenous information transmitted by commodity prices. More precisely, we state that a vector of commodity prices $p \in \mathbb{R}_+^{L \times S^*}$ reveals information which allows to distinguish between states of nature s and s' if and only if $\frac{p_s}{p_s \cdot v} \neq \frac{p_{s'}}{p_{s'} \cdot v}$, for any $v \in \mathbb{R}_{++}^L$. In other words, the information obtained from commodity prices cannot be a function of the numeraire that was chosen to measure prices. Thus, two vectors of commodity prices p and p' reveal the same information about the states of nature that was realized when, for each pair $(s, s') \in S \times S$ and $v \in \mathbb{R}_{++}^L$, $\frac{p_s}{p_s \cdot v} \neq \frac{p_{s'}}{p_{s'} \cdot v}$ if and only if $\frac{p'_s}{p'_s \cdot v} \neq \frac{p'_{s'}}{p'_s \cdot v}$.

The sequential process of adapting her private information happens at the same time agents adapt their individual state dependent preferences. We assume that any $i \in I$ has a utility function $V^i : \mathbb{R}_+^{L \times S^*} \times \mathbb{R}_+^{L \times S^*} \to \mathbb{R}$ such that, $V^i(p, x) = V^i(p', x)$ for any pair of prices (p, p') that reveal the same information. Given prices $p \in \mathcal{P}$, let $\tau(p)$ be the partition of S generated by the information revealed by commodity prices. Then, the final private information of agent i is given by $\mathbb{P}^i \vee \tau(p)$.

Each agent $i \in I$ selects her consumption by choosing an informational and budgetary compatible allocation $(x_s^i; s \in S^*) \in \mathbb{R}_+^{L \times S^*}$, implemented through a financial position $\theta^i = (\theta_j^i; j \in J^i) \in \mathbb{R}^{J^i}$. More precisely, given prices $(p, q) \in \mathcal{P} \times \mathbb{R}_+^J$, the objective of an agent $i \in I$ is to maximize his utility function $V^i(p, \cdot)$ by choosing an allocation in his *choice set*, defined as the collection of vectors $(x^i, \theta^i) \in \mathbb{E}^i := \mathbb{R}_+^{L \times S^*} \times \mathbb{R}^{J^i}$ such that $(x_s^i; s \in S)$ is $\mathbb{P}^i \vee \tau(p)$ -measurable, and

$$\begin{aligned} p_0 x_0^i + \sum_{j \in J^i} q_j \theta_j^i &\leq p_0 w_0^i, \\ p_s x_s^i &\leq p_s w_s^i + \sum_{j \in J^i} R_{s,j} \theta_j^i, \ \forall s \in S. \end{aligned}$$

[8] Given a partition $\mathbb{P}$ of S, a vector $(v_s; s \in S)$ is $\mathbb{P}$-measurable if $v_s = v_{s'}$ for any pair of states of nature s and s' which belongs to the same element of the partition $\mathbb{P}$.

The collection of allocations $(x^i,\theta^i)\in\mathbb{E}^i$ that satisfy budget constraints above is denoted by $B^i(p,q)$, while the collection of vectors $(x^i,\theta^i)\in\mathbb{E}^i$ for which $(x^i_s;s\in S)$ is $\mathbb{P}^i\vee\tau(p)$-measurable is denoted by $\mathcal{I}^i(p)$. Therefore, given $(p,q)\in\mathcal{P}\times\mathbb{R}^J_+$, the choice set of agent $i\in I$ is $B^i(p,q)\cap\mathcal{I}^i(p)$.

Definition. *An equilibrium with endogenous differential information and restricted financial participation is given by prices* $(\overline{p},\overline{q})\in\mathcal{P}\times\mathbb{R}^J_+$ *and allocations* $\left((\overline{x}^i,\overline{\theta}^i);i\in I\right)\in\prod_{i\in I}\mathbb{E}^i$ *such that,*

(i) For any agent $i\in I$, $\left(\overline{x}^i,\overline{\theta}^i\right)\in\mathbb{E}^i$ *maximizes the utility function* $V^i(\overline{p},\cdot)$ *among the allocations in the choice set* $B^i(\overline{p},\overline{q})\cap\mathcal{I}^i(\overline{p})$.

(ii) The following markets clearing conditions hold,

$$\sum_{i\in I}\left(\overline{x}^i_s-w^i_s\right)=0,\quad\forall s\in S^*;\qquad\qquad\sum_{i\in I(j)}\overline{\theta}^i_j=0,\ \forall j\in J,$$

where, for any asset $j\in J$, $I(j):=\{i\in I:j\in J^i\}$.

The following remark illustrates the difficulties that may appear in order to ensure equilibrium existence in a model where differential information is endogenous, since choice set correspondences do not necessarily have a closed graph.

Remark. Fix an agent $i\in I$ that is not fully informed (i.e., $\mathbb{P}^i\neq\{\{s\};s\in S\}$), and consider a sequence of commodity prices $\{p_n\}_{n\geq1}\subset\mathcal{P}$ that converges to $\overline{p}$. Suppose, that for any $n\in\mathbb{N}$, there is a partition $\mathbb{Q}$ such that $\mathbb{Q}=\mathbb{P}^i\vee\tau(p_n)$. Also, assume that $\mathbb{Q}$ is strictly finer than $\mathbb{P}^i\vee\tau(\overline{p})$.

Let $\theta^i=0$ and $x^i=(w^i_0,(\alpha_s w^i_s;s\in S))$, where $(\alpha_s;s\in S)\in(0,1)^S$ is $\mathbb{Q}$-measurable but not $\mathbb{P}^i$-measurable. Then, independently of $q\in\mathbb{R}^J_+$, the plan (x^i,θ^i) belongs to $B^i(p_n,q)\cap\mathcal{I}^i(p_n)$, for any $n\geq1$. However, $(x^i,\theta^i)\notin B^i(\overline{p},q)\cap\mathcal{I}^i(\overline{p})$, since this plan is only $\mathbb{Q}$-measurable. Therefore, the choice set correspondence does not have a closed graph. □

In order to guarantee that an equilibrium exists, we need to recover the closed graph property of the choice set. For this reason, we endogenize the information compatibility constraint. That is, we will impose hypotheses on preferences that allow us to obtain the compatibility between consumption and final information without the need to restrict individual allocations.

3. Existence of equilibrium

A priori, there is no reason to assume that agents have the same preferences for consumption at different states of nature. However, when a consumer does not distinguish the realization of the uncertainty, it is natural to assume that she cannot take advantage of the heterogeneity of her tastes in states of nature that she does not recognize. Since agents are able to forecast spot commodity prices, we will assume that they have informational dependent preferences and make decisions considering the endogenous information obtained after receiving the public signal generated by commodity prices.

Assumption A1. *Given $i \in I$, for any $s \in S$ there are continuous functions $\pi_s^i : \mathcal{P} \to (0,1)$ and $u_s^i : \mathcal{P} \times \mathbb{R}_+^L \times \mathbb{R}_+^L \to \mathbb{R}$ such that, $V^i(p,x) = \sum_{s\in S} \pi_s^i(p) u_s^i(p, x_0, x_s)$.*

In addition, for any price $p \in \mathcal{P}$, we have that: (i) $\sum_{s\in S} \pi_s^i(p) = 1$; (ii) $(u_s^i(p,\cdot))_{s\in S}$ are strictly concave and strictly increasing functions; and (iii) $u_s^i(p,\cdot) = u_{s'}^i(p,\cdot)$, when s and s' are undistinguishable under $\mathbb{P}^i \vee \tau(p)$.

Notice that the assumption above, ensures that for any $z \in \mathbb{R}_+^L \times \mathbb{R}_+^L$, vectors $(u_s^i(p,z); s \in S)_{i\in I}$ are $\mathbb{P}^i \vee \tau(p)$-measurable. That is, agents cannot take advantage of heterogeneous tastes at states of nature which they cannot distinguish. Indeed, changes in prices generate effects over state-contingent utility levels only when the revealed information refine initial knowledge. Thus, as in the model by Pollak (1977), preferences are influenced by relative rather than absolute prices.

Since Assumption (A1) allows state contingent probabilities $(\pi_s^i(p); s \in S)$ to depend on commodity prices, our framework may also capture the informational effects of prices on agent's beliefs about the state of nature that will be realized.

Example 1. Suppose that $S = \{a, b\}$ and, for some agent $i \in I$, the utility function is given by $V^i(p,x) = 0.5u_a^i(p, x_0, x_a) + 0.5u_b^i(p, x_0, x_b)$, where

$$\begin{aligned} u_a^i(p, x_0, x_a) &= \Omega(p) v_a(x_0, x_a) + (1 - \Omega(p)) u(x_0, x_a), \\ u_b^i(p, x_0, x_b) &= \Omega(p) v_b(x_0, x_b) + (1 - \Omega(p)) u(x_0, x_b), \end{aligned}$$

and $v_a, v_b, u : \mathbb{R}_+^L \times \mathbb{R}_+^L \to \mathbb{R}$ are continuous, strictly concave and strictly monotonic functions. Also, the continuous function $\Omega : \mathcal{P} \to \mathbb{R}_+$ is given by $\Omega(p) = 0$ when prices do not reveal information (i.e. $\{p_a, p_b\}$ are collinear), and $\Omega(p) > 0$ in another case. With this characterization, it follows that the utility function V^i satisfies Assumption (A1).

Assume that agent i is initially uninformed on the states of nature, i.e., $\mathbb{P}^i = \{a, b\}$. Then, when he recognizes the states of nature through the information revealed by prices, he can choose different bundles of consumption motivated by the heterogeneity of preferences, which is captured by functions v_a and v_b in conjunction with the fact that $\Omega(p) \neq 0$. □

The following result ensures that, under Assumption (A1), and without need to restrict consumption allocations on choice sets, optimality of agents decisions endogenizes the compatibility between consumption and final information.

Proposition. *Given prices* $(p,q) \in \mathcal{P} \times \mathbb{R}^J_+$*, suppose that* $((x^i_s; s \in S^*), \theta^i) \in \mathbb{E}^i$ *is an optimal choice for agent* $i \in I$ *on* $B^i(p,q)$*. If Assumption (A1) holds, then* $(x^i_s; s \in S)$ *belongs to* $\mathcal{I}^i(p)$.

Proof. Suppose that states of nature s and s' are in the same element of $\mathbb{P}^i \vee \tau(p)$. Since $p \in \mathcal{P}$, we have that $p_s = p_{s'}$. Moreover, as $\mathbb{P}^i$ contains the information revealed by the payments of assets in J^i, $p_s w^i_s + \sum_{j \in J^i} R_{s,j}\theta^i_j = p_{s'} w^i_{s'} + \sum_{j \in J^i} R_{s',j}\theta^i_j$.

Thus, for any $\lambda \in [0,1]$, the bundle $\lambda x^i_s + (1-\lambda)x^i_{s'}$ is budget feasible at both states of nature, s and s'. Using Assumption (A1), it follows from individual optimality that, $u^i_{s'}(p, x^i_0, x^i_s) = u^i_s(p, x^i_0, x^i_s) \geq u^i_s(p, x^i_0, x^i_{s'})$ and $u^i_s(p, x^i_0, x^i_{s'}) = u^i_{s'}(p, x^i_0, x^i_{s'}) \geq u^i_{s'}(p, x^i_0, x^i_s)$. Therefore, for any $k \in \{s, s'\}$, $u^i_k(p, x_0, x_s) = u^i_k(p, x_0, x_{s'})$. This implies that, if $x^i_s \neq x^i_{s'}$, then the strictly concavity of the utility function ensures that agent i can improve his utility level choosing the bundle $\tilde{x} := 0.5x^i_s + 0.5x^i_{s'}$ in both states of nature. A contradiction.

Thus, the optimal plan $(x^i_s; s \in S^*)$ is informational compatible with $\mathbb{P}^i \vee \tau(p)$. □

Different to the traditional two-period general equilibrium model, the existence of incomplete financial participation does not allow us to compactify the space of prices—normalizing (p_0, q) to belong on $\Delta := \{\nu \in \mathbb{R}^L_+ \times \mathbb{R}^J_+ : \|\nu\|_\Sigma = 1\}$—and, at the same time, ensure that budget set correspondences has a non-empty interior. However, these two properties are relevant as they allow us to obtain a competitive equilibrium for our economy as a Cournot-Nash equilibrium of a generalized game where strategies and prices are truncated. Therefore, we guarantee the non-emptiness of the interior of budget set correspondences normalizing commodity prices to belong to $\mathcal{P}$. For these reasons, we need to find endogenous upper bounds for asset prices.

Thus, we impose the following assumption on the impatience of agents, that was previously addressed by Seghir and Torres-Martínez (2011) in their financial model with incomplete financial access to credit opportunities.

Assumption A2. *For any* $i \in I$*, given* $\sigma \in (0,1)$ *there is* $r_\sigma : \mathcal{P} \times \mathbb{R}^{L \times S^*}_+ \to \mathbb{R}^L_+$*, continuous in* $p \in \mathcal{P}$*, that satisfies*

$$V^i(p, x_0 + r_\sigma(p,x), (\sigma x_s; s \in S)) > V^i(p, (x_s; s \in S^*)), \quad \forall p \in \mathcal{P},\ \forall (x_s; s \in S^*) \in \mathbb{R}^{L \times S^*}_+.$$

Notice that the assumption above is satisfied for any utility function that is unbounded on first period consumption. However, (A2) also holds for a great variety of bounded utility functions.

As we have pointed out above, we do not intend to impose any kind of financial survival assumption to ensure the existence of equilibrium. Indeed, only when agents do not have any access to some financial instruments (i.e., for some $i \in I$, $J^i \neq J$) we can guarantee that financial markets have relevant information not obtained by some agents. For this reason, to prove that an equilibrium exists, we need to extend the arguments of Seghir and Torres-Martínez (2011) to allow for restrictions on investment opportunities.

Theorem. *Under Assumptions (A1) and (A2), there exists an equilibrium for the economy with endogenous differential information and incomplete financial participation.*

Proof. See the Appendix.

The following examples offer some insights into price informativeness and information compatibility, captured by the model.

Example 2. Consider an economy with two commodities and utility functions given by

$$U^i((x_s; s \in S^*)) = \sum_{s\in S} \pi_s^i \left(x_{0,1}^{\beta} x_{0,2}^{1-\beta} + x_{s,1}^{\beta} x_{s,2}^{1-\beta} \right), \quad \forall i \in I,$$

where $\beta \in (0,1)$ is the same for all agents. Then, Assumption (A1)-(A2) hold. Also, first-order conditions of consumer's i problem at state $s \in S$ would imply that, at any equilibrium price $\overline{p}_s$,

$$\frac{\overline{p}_{s,1}}{\overline{p}_{s,2}} = \frac{\beta}{1-\beta} \frac{W_{s,2}}{W_{s,1}},$$

where $W_{s,l} = \sum_{i\in I} w_{s,l}^i$. Suppose that there is an uninformed agent $i_0 \in I$ (i.e. $\mathbb{P}^{i_0} = \{S\}$). Then, equilibrium prices are *non-informative*[9] if, and only if, the relative degree of commodity scarcity is constant at the second period, $\frac{W_{s,2}}{W_{s,1}} = \frac{W_{s',2}}{W_{s',1}}$, $\forall (s,s') \in S \times S$, which is a restrictive hypothesis. Thus, for any economy in which this condition does not hold, any equilibrium price will reveal information (at least for the uninformed agent i_0). □

Example 3. In this example we will illustrate the importance of the compatibility between the initial information and the information revealed by the asset that an agent can trade. For simplicity, consider an economy with only one commodity, three states of nature at $t = 1$, denoted by $\{u, m, d\}$, and two agents who only receive utility for consumption in the second period. Also, they do not have any initial endowment at $t = 0$. Thus, utility functions and endowments are given by

$$U^1(x_u, x_m, x_d) = \sqrt{2}\sqrt{x_u} + \sqrt{x_m} + \sqrt{x_d}, \qquad (w_u^1, w_m^1, w_d^1) = (3,3,3);$$

$$U^2(x_u, x_m, x_d) = \sqrt{x_u} + \sqrt{x_m} + \sqrt{2}\sqrt{x_d}, \qquad (w_u^2, w_m^2, w_d^2) = (3,3,3).$$

[9]That is, prices are measurable with respect to the coarse information partition

There are two Arrow securities in the economy. One of them has a unitary price q_1 and promises to deliver one unit of the commodity at state of nature $s = u$. The other makes a contingent payment of one unit of the commodity at $s = d$, and is negotiated for a unitary price q_2.

If there is a complete financial participation, the first period budget constraint of agent $i \in \{1, 2\}$ is given by $q_1 z^i_1 + q_2 z^i_2 = 0$, where z^i_j denotes the position of agent i on asset $j \in \{1, 2\}$.

Assume that unitary prices are given by $\overline{q}_1 = \overline{q}_2 = 1$. Then, the allocations

$$(z^1_1, z^1_2, x^1_u, x^1_m, x^1_d) = (1, -1, 4, 3, 2), \qquad (z^2_1, z^2_2, x^2_u, x^2_m, x^2_d) = (-1, 1, 2, 3, 4).$$

constitute an equilibrium for the economy.

We argue that, if agents are not fully informed—that is, they do not internalize the information revealed by asset payments— the implementation of this equilibrium allocation may not be credible. For instance, assume that $\mathbb{P}^1 = \{\{u\}, \{m, d\}\}$ and $\mathbb{P}^2 = \{\{u, m\}, \{d\}\}$. In order to pay his debt, it is required that agent $i = 1$ observes that the state of nature $s = d$ was realized. This would be impossible to accomplish as commodity prices do not communicate information, and there is no other financial signal which allows recognition between states m and d. Analogously, to pay his debt, it is required that agent $i = 2$ observes some signal that allows him to distinguish between u and m, which is an impossible task to accomplish given the financial structure. □

4. Concluding remarks

In this paper we extend the model of competitive markets with differential information introduced by Radner (1968), allowing for sequential trade of commodities and incomplete financial participation. Agents trade assets in financial markets, buy commodities in spot markets and receive signals allowing them to improve their private information about the realization of the state of nature.

The information about the realization of uncertainty has real effects over the agent's capability to implement heterogeneous preferences across states of nature. Equilibrium existence is obtained without imposing any compatibility requirement between consumption and information. The measurability of optimal bundles is a consequence of the informational-dependent nature of individuals objective functions, since there are no gains for consumption heterogeneity in states of nature that are undistinguishable.

Our model allows agents to obtain information through the variability of payments in financial markets. Thus, individuals obtain all the information revealed by the awareness conveyed by securities that they can trade. However, there is an incomplete access to (and knowledge of) financial instruments available in the economy. For this reason, our model is compatible with higher degrees of market completeness even when some agents have lower levels of information. Essentially, it is a consequence of the limited participation that uninformed individuals have on financial markets.

However, to allow assets for information transmission to some agents only, we need to ensure that equilibrium exists without the need to require any kind of financial survival restriction. Thus, we extend the model of credit constrained markets of Seghir and Torres-Martínez (2011) to allow for an incomplete access to investment opportunities.

Appendix: Proof of Theorem 1

To prove equilibrium existence, we first define a generalized game in which agents maximize utility functions in truncated budget sets. Auctioneers choose prices in order to maximize the value of the excess of demand in commodity and financial markets. We prove that this generalized game has a Cournot-Nash equilibrium and that also, when the upper bounds on allocations are high enough, any equilibrium of the generalized game will be an equilibrium of our economy.

The generalized game $\mathcal{G}(Q, X, \Theta)$. Given any vector $(Q, X, \Theta) \in \mathbb{R}^3$, we define a game characterized by the following set of players and strategies.

Set of players. There is a finite set of players constituted by,

(i) The set of agents of the economy, I.

(ii) An auctioneer, $h(s)$, for each $s \in S^*$.

We denote the set of players by $H = I \cup H(S^*)$ where $H(S^*) := \{h(s) : s \in S^*\}$.

Sets of strategies. Given $\overline{W} := \max\limits_{(s,l)\in S^*\times L} \sum_{i\in I} w^i_{s,l}$, define for any $i \in I$,

$$K^i(X, \Theta) = [0, X]^L \times [0, 2\overline{W}]^{S\times L} \times [-\Theta, \Theta]^{J^i},$$

and, for any $s \in S^*$, let $\mathcal{P}_s = \{p \in \mathbb{R}^L_+ : p \cdot \zeta = 1\}$. The set of strategies for the players in the generalized game, $(\overline{\Gamma}^h; h \in H)$, are given by,

(i) For each $h \in I$, $\overline{\Gamma}^h = K^h(X, \Theta)$.

(ii) For $h = h(0)$, $\overline{\Gamma}^h = \mathcal{P}_0 \times [0, Q]^{\#J}$

(iii) For $h = h(s)$, with $s \in S$, $\overline{\Gamma}^h = \mathcal{P}_s$.

For simplicity, let $\eta^h = \left(x^h, \theta^h\right) \in \overline{\Gamma}^h$ be a generic vector of strategies for a player $h \in I$; (p_0, q) will denote a generic strategy for the player $h(0)$; and p_s a generic strategy for a player $h(s)$, with $s \in S$. Finally, let $\overline{\Gamma} = \prod_{h\in H} \overline{\Gamma}^h$ be the space of strategies of $\mathcal{G}(Q, X, \Theta)$. A generic element of $\overline{\Gamma}$ is denoted by (p, q, η), where $\eta := \left(\eta^h; h \in I\right)$ is a generic element of $\prod_{i\in I} \overline{\Gamma}^i$.

Admissible strategies. Strategies effectively chosen for players depend on the actions taken by other players, through a correspondence of admissible strategies $\phi^h : \overline{\Gamma}_{-h} \twoheadrightarrow \overline{\Gamma}^h$, where $\overline{\Gamma}_{-h} = \prod_{h'\neq h} \overline{\Gamma}^{h'}$. Let $(p, q, \eta)_{-h}$ be a generic element of $\overline{\Gamma}_{-h}$. We suppose that,

(i) If $h \in I$, $\phi^h\left[(p,q,\eta)_{-h}\right] = B^h(p,q) \cap \overline{\Gamma}^h$.
(ii) If $h \in H(S^*)$, $\phi^h\left[(p,q,\eta)_{-h}\right] = \overline{\Gamma}^h$.

Objective functions. Each player is also characterized by an objective function $F^h : \overline{\Gamma}^h \times \overline{\Gamma}_{-h} \to \mathbb{R}_+$. We assume that,

(i) When $h \in I$ and $\eta^h = \left(x^h, \theta^h\right) \in \overline{\Gamma}^h$, then $F^h\left(\eta^h; (p,q,\eta)_{-h}\right) = V^i\left(p, x^h\right)$.
(ii) If $h = h(0)$ and $(p,q) \in \overline{\Gamma}^h$, then

$$F^h\left((p_0,q);(p,q,\eta)_{-h}\right) := p_0 \sum_{i \in I}\left(x_0^i - w_0^i\right) + \sum_{i \in I}\sum_{j \in J^i} q_j \theta_j^i.$$

(iii) If $h(s) \in H(S^*) \setminus \{h(0)\}$ and $p_s \in \overline{\Gamma}^h$, then $F^h\left(p_s;(p,q,\eta)_{-h}\right) := p_s \sum_{i \in I}\left(x_s^i - w_s^i\right)$.

We define the correspondence of optimal strategies for each $h \in H$, $\Psi^h : \overline{\Gamma}_{-h} \twoheadrightarrow \overline{\Gamma}^h$ as

$$\Psi^h\left((p,q,\eta)_{-h}\right) := \underset{y \in \phi^h\left((p,q,\eta)_{-h}\right)}{\arg\max} F^h\left(y;(p,q,\eta)_{-h}\right).$$

Finally, let $\Psi : \overline{\Gamma} \twoheadrightarrow \overline{\Gamma}$ be the correspondence of optimal game response, which is given by $\Psi(p,q,\eta) = \prod_{h \in H} \Psi^h\left((p,q,\eta)_{-h}\right)$.

Definition. *A Cournot-Nash equilibrium for the generalized game $\mathcal{G}(Q,X,\Theta)$ is given by a strategy profile $(\overline{p},\overline{q},\overline{\eta}) \in \overline{\Gamma}$ such that, $(\overline{p},\overline{q},\overline{\eta}) \in \Psi(\overline{p},\overline{q},\overline{\eta})$.*

In order to prove the existence of equilibrium in the generalized game, we need some properties of the admissible strategy correspondence which the following lemma provides.

Lemma 1. *For any $h \in H$, ϕ^h is continuous and has non-empty, compact, and convex values.*

Proof. For each player $h \in H(S^*)$, the correspondence of admissible strategies is constant and, therefore, it is continuous and non-empty. Also, by definition, its values are compact and convex.

On the other hand, for each player $h \in I$, it follows, from the definition of the budget set, that the correspondence of admissible strategies ϕ^h has non-empty, compact and convex values. Since the graph of this correspondence is closed, we obtain upper hemicontinuity. To assure the lower hemicontinuity of ϕ^h, we consider the correspondence $\mathring{\phi}^h\left((p,q,\eta)_{-h}\right) := int_{K^h(X,\Theta)}\, B^h(p,q)$, which associates to a vector of commodity and asset prices the set of allocations in $K^h(X,\Theta)$ that satisfy all the budget restrictions of agent h as strict inequalities. Note that, this correspondence has non-empty values and open graph. Therefore, it is lower hemicontinuous. We know that the closure of $\mathring{\phi}^h\left((p,q,\eta)_{-h}\right)$, which is equal to $\phi^h\left((p,q,\eta)_{-h}\right)$, is also lower hemicontinuous. Therefore, correspondences of admissible strategies $(\phi^h; h \in I)$ are continuous. □

Lemma 2. *Under (A1), the set of Cournot-Nash equilibria of* $\mathcal{G}(Q, X, \Theta)$ *is non-empty.*

Proof. By Assumption (A1), each objective function in the game is continuous in all variables and quasi-concave in its own strategy. Also, the sets of strategies are non-empty, compact and convex. By Lemma 1, admissible correspondence is continuous with non-empty, convex and compact-values. Thus, we can apply Berge's Maximum Theorem to assure that, for each player $h \in H$ the correspondence of optimal strategies, Ψ^h, is upper hemicontinuous with non-empty, convex and compact values. Therefore, the correspondence Ψ has closed graph with non-empty, compact and convex values. Applying Kakutani's Fixed Point Theorem to Ψ we conclude the proof. □

We will prove that, for vectors $(Q, X, \Theta) \in \mathbb{R}^3_+$ for which coordinates are high enough, any equilibrium of the generalized game is an equilibrium for our economy. However, we need to previously find endogenous upper bounds for equilibrium variables.

Lemma 3. *For each* $s \in S$*, fix a vector* $(p_s, w_s, x_s) \in \mathcal{P}_s \times \mathbb{R}^L_+ \times \mathbb{R}^L_+$*, with* $x_s < \overline{W}$*. Then, there exists* $A > 0$ *such that, any allocations* $(\kappa_j; j \in J) \in \mathbb{R}^J$ *satisfying*

$$p_s x_s = p_s w_s + \sum_{j \in J} R_{s,j} \kappa_j, \ \forall s \in S;$$

belongs on $[-A, A]^{\#J+1}$*. Also, A only depends on* $((\overline{W}, w_s, R_{s,j}); (s, j) \in S \times J)$*.*

Proof. Note that, as S (respectively, J) is a finite set, by abusing of the notation and identifying it with $\{1, \ldots, S\}$ (respectively, $\{1, \ldots, J\}$) we can rewrite the conditions in the statement of the Lemma in a matricial form:

$$\begin{bmatrix} p_1(x_1 - w_1) \\ \vdots \\ p_S(x_S - w_S) \end{bmatrix} = \begin{bmatrix} R_{1,1} & \cdots & R_{1,J} \\ \vdots & \ddots & \vdots \\ R_{S,1} & \cdots & R_{S,J} \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_J \end{bmatrix}$$

Since there is no redundant assets in the economy, we have that $J \leq S$. Moreover, we can find a non-singular sub-matrix of dimension $J \times J$. Specifically, we may assume, without loss of generality, that this matrix is given by

$$B = \begin{vmatrix} R_{1,1} & \cdots & R_{1,J} \\ \vdots & \vdots & \vdots \\ R_{J,1} & \cdots & R_{J,J} \end{vmatrix}.$$

Thus, we have that

$$\begin{bmatrix} p_1(x_1 - w_1) \\ \vdots \\ p_J(x_J - w_J) \end{bmatrix} = B \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_J \end{bmatrix}$$

By Cramer's Rule,

$$\kappa_j = \frac{det(B(y,j))}{\det(B)}, \ \ \forall j \in \{1,\dots,J\},$$

where $y = (p_1(x_1 - w_1), \dots, p_J(x_J - w_J))$ and $B(y,j)$ is the matrix obtained by change, in the matrix B, the j-ith column for the vector y. Since (i) the determinant is a continuous function; (ii) the vector y depends continuously of $((p_s, x_s); s \in S)$; and (iii) vectors $((p_s, x_s, w_s); s \in S)$ are in a compact space, it follows that vector $(k_j; j \in J)$ is bounded, independently of the value of $((p_s, x_s, w_s); s \in S)$. Thus, there exists $A > 0$ which satisfies the conditions of the lemma and depends on $((\overline{W}, w_s, R_{s,j}); (s,j) \in S \times J)$. □

Following the notation of the previous lemma, let $\overline{\Theta} := 2A$.

The next two lemmas are used to prove that equilibrium asset prices of the generalized game are uniformly bounded. For convenience of notations, let $W_0 = (W_{0,l}; l \in L)$ be the vector of aggregated physical resources at $t = 0$, where $W_{0,l} := \sum_{i \in I} w^i_{0,l}$.

Lemma 4. *Under Assumptions (A1)-(A2), fix $(\overline{p}, \overline{q}) \in \mathcal{P} \times \mathbb{R}^J_+$ and suppose that, for any agent $i \in I$, there is an optimal solution $(\overline{x}^i, \overline{\theta}^i) \in \overline{\Gamma}^i$ for his individual problem such that, $\overline{x}^i_0 \leq W_0$ and $\overline{x}^i_{s,l} \leq 2\overline{W}$, $\forall (s,l) \in S \times L$. Then, there exists $\overline{Q} > 0$, independent of prices, such that $\max_{j \in J} \overline{q}_j < \overline{Q}$.*

Proof. Fix $j \in J$. Suppose that an agent $i \in I$ for which $j \in J^i$ borrows a quantity $\tilde{\theta}_j > 0$ of asset j such that $R_{s,j}\tilde{\theta}_j \leq \mu := \frac{\min_{(k,l,h) \in S \times L \times I} w^h_{k,l}}{2}$, for any $s \in S$.[10] This position on asset j reports a quantity of resources which allow agent i to consume at the first period the bundle $w^i_0 + (\overline{q}_j \tilde{\theta}_j)\zeta$ and, therefore,

$$V^i\left(\overline{p}, w^i_0 + (\overline{q}_j \tilde{\theta}_j)\zeta, (0.5 w^i_s; s \in S)\right) \leq V^i(\overline{p}, \overline{x}^i) < V^i\left(\overline{p}, W_0, (2\overline{W}(1,\dots,1))_{s \in S}\right).$$

On the other hand, Assumption (A2) guarantees that there exists $\overline{r}(\overline{p}) \in \mathbb{R}^L_+$ such that,

$$V^i\left(\overline{p}, W_0, (2\overline{W}(1,\dots,1))_{s \in S}\right) < V^i(\overline{p}, w^i_0 + \overline{r}(\overline{p}), (0.5 w^i_s; s \in S)).$$

Indeed, following the notation of Assumption (A2), the inequality above follows from

$$\overline{r}(\overline{p}) = \overline{r}_{\tilde{\sigma}}\left(\overline{p}, (W_0, (2\overline{W}(1,\dots,1))_{s \in S})\right) + W_0 - w^i_0 \in \mathbb{R}^L_+,$$

[10]Notice that, by definition, $\tilde{\theta}_j$ depends only on primitive parameters of the economy (endowments and unitary financial payments).

where $\tilde{\sigma} \in (0,1)$ is chosen to satisfiy $2\overline{W}\tilde{\sigma} < \mu$.

We conclude that,

$$\overline{q}_j < Q_j(\overline{p}) := \frac{\|\overline{r}(\overline{p})\|_\Sigma}{\tilde{\theta}_j \|\zeta\|_\Sigma}.$$

Moreover, the upper bound $Q_j(p)$ is well defined for any $p \in \mathcal{P}$ and, it follows from Assumption (A2), that it varies continuously with commodity prices. Thus, the function $Q : \mathcal{P} \to \mathbb{R}$ defined by $Q(p) = \max_{j\in J} Q_j(p)$ is continuous. Since $\mathcal{P}$ is compact, we conclude that there exists $\overline{Q} > 0$ such that, $\max_{j\in J} \overline{q}_j < \overline{Q}$. □

We define $\overline{X} = 2(1 + \overline{Q})\overline{W}$.

Note that, for any $X > \overline{X}$ and $Q > \overline{Q}$, in the associated generalized game $\mathcal{G}(Q, X, \Theta)$ any player $h \in I$ may demand in the first period the bundle used in the proof of Lemma 4. Thus, in this type of generalized game, the existence of an optimal plan satisfying the conditions of lemma above will imply that the unitary prices of assets are bounded from above by $\overline{Q}$.

The existence of equilibria in our economy is a consequence of the following result.

Lemma 5. *Under Assumptions (A1)-(A2), if $(Q, X, \Theta) \gg (\overline{Q}, \overline{X}, \overline{\Theta})$, then every Cournot-Nash equilibrium for $\mathcal{G}(Q, X, \Theta)$ is an equilibrium of the original economy.*

Proof. Let $\left(\overline{p}, \overline{q}, (\overline{\eta}^i; i \in I)\right)$, where $\overline{\eta}^i = \left(\overline{x}^i, \overline{\theta}^i\right) \in \overline{\Gamma}^i$, be a equilibrium for the generalized game $\mathcal{G}(Q, X, \Theta)$, with $(Q, X, \Theta) \gg (\overline{Q}, \overline{X}, \overline{\Theta})$.

Step I: Market feasibility. Aggregating agent's first period budget constraints we have,

$$\overline{p}_0 \sum_{i\in I} \left(\overline{x}_0^i - w_0^i\right) + \sum_{i\in I} \sum_{j\in J^i} \overline{q}_j \overline{\theta}_j^i \leq 0.$$

It follows that, if $\sum_{i\in I} \left(\overline{x}_{0,l}^i - w_{0,l}^i\right) > 0$, then the auctioneer $h(0)$ will choose the greater price for this good, $\overline{p}_l = 1$, and zero prices for the other goods and assets, making his objective function positive, which contradicts the inequality above. Therefore, $\sum_{i\in I} \overline{x}_0^i \leq \sum_{i\in I} w_0^i < W_0$. Analogously, if $\sum_{i\in I(j)} \overline{\theta}_j^i > 0$, then the auctioneer $h(0)$ would choose the maximum price possible for this asset, i.e. $\overline{q}_j = Q > \overline{Q}$, which is a contradiction with the result of Lemma 4. Thus, for any $j \in J$, $\sum_{i\in I(j)} \overline{\theta}_j^i \leq 0$.

Since first period consumption is bounded from above by the aggregate endowment, which is less than X, it follows that budget constraints at $t = 0$ are satisfied with equality. Hence, the auctioneer $h(0)$ has an optimal value equal to zero. As a consequence, if $\sum_{i\in I} \left(\overline{x}_{0,l}^i - w_{0,l}^i\right) < 0$, the auctioneer $h(0)$ would choose a zero price for the good l, a contradiction with the strictly monotonicity of preferences (Assumption (A1)). Therefore, $\sum_{i\in I} \overline{x}_0^i = W_0$. Furthermore, if $\sum_{i\in I(j)} \overline{\theta}_j^i < 0$, the auctioneer would choose $\overline{q}_j = 0$, a contradiction with the strictly monotonicity of preferences. Then, market feasibility conditions hold at $t = 0$ in both physical and financial markets.

Using the market feasibility of $\left(\left(\overline{x}^i, \overline{\theta}^i\right); i \in I\right)$ at $t = 0$, and aggregating budget constraints at $s \in S$, we obtain that $\overline{p}_s \sum_{i \in I} \left(\overline{x}^i_s - w^i_s\right) \leq 0$. Therefore, analogous arguments to those made above ensure that $\sum_{i \in I} \left(\overline{x}^i_s - w^i_s\right) \leq 0$. This last property guarantees that budget constraints are satisfied as an equality in the state of nature s. Finally, if $\sum_{i \in I} \left(\overline{x}^i_{s,l} - w^i_{s,l}\right) < 0$, then the auctioneer $h(s)$ would choose a zero price for the good $l \in L$, which contradicts individual optimality under strictly monotonic preferences. We conclude that market feasibility also holds at each state of nature $s \in S$.

Step II. Optimality of individual allocations. Since market feasibility holds in physical markets, it follows that $\overline{x}^i_{0,l} < X$ and $\overline{x}^i_{s,l} < 2\overline{W}$, for any $(i, s, l) \in I \times S \times L$. Using Lemma 3, we have that for any $i \in I$ and $j \in J^i$, $|\theta^i_j| < \Theta$. Thus, for any $i \in I$, $\overline{\eta}^i$ belongs on the interior of $K^i(X, \Theta)$.

Suppose that there exists another allocation $\eta^i \in \mathbb{R}^{L \times S^*}_+ \times \mathbb{R}^{J^i}$ such that $V^i(\overline{p}, \eta^i) > V^i(\overline{p}, \overline{\eta}^i)$. Since for $\lambda \in (0, 1)$ sufficiently small, $\eta^i(\lambda) := \lambda \eta^i + (1 - \lambda)\overline{\eta}^i \in K^i(X, \Theta)$, the strictly concavity of $V^i(\overline{p}, \cdot)$ implies that $V^i(\overline{p}, \eta^i(\lambda)) > V^i(\overline{p}, \overline{\eta}^i)$, a contradiction with the optimality of $\overline{\eta}^i \in \overline{\Gamma}^i$. Therefore, for any $\eta^i \in \mathbb{R}^{L \times S^*}_+ \times \mathbb{R}^{J^i}$, $V^i(\overline{p}, \eta^i) \leq V^i(\overline{p}, \overline{\eta}^i)$, which proves the optimality of $\overline{\eta}^i \in B^i(\overline{p}, \overline{q})$ among the allocations in the agent i's budget set. Notice that, as was proved in Section 3, informational compatibility of consumption allocations follows from Assumption (A1). □

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Department of Economics,University of Chile
Diagonal Paraguay 257, Santiago, Chile
E-mail address: secea@fen.uchile.cl

RGEA, Facultad de Económicas, Universidad de Vigo
Lagoas Marcosende 36310, Vigo, España
E-mail address: cherves@uvigo.es

Department of Economics,University of Chile
Diagonal Paraguay 257, Santiago, Chile
E-mail address: juan.torres@fen.uchile.cl