Dealership Equilibria in Oligopoly

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Abstract

This paper investigates why oligopolistic manufacturers may choose to sell their products through an independent dealer rather than directly to final consumers. In our model, manufacturers can observe neither the realised demand nor the sales services provided by the dealer and must incur a monitoring cost to ascertain the sales services provided . Manufacturers choose a permissible discount, followed by a price-quantity target to be implemented by the dealer. We show that if the monitoring cost parameter is too high, the firm might deprive the dealer of any decision power by behaving like a Bertrand competitor. This is akin to a no-dealership equilibrium. For sufficiently low values of a monitoring cost parameter, the firm chooses the degree of flexibility for their dealer, and the equilibrium market outcome ranges from Cournot to Bertrand contingent on the parameter.

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1 Introduction

Virtually every good sold in a modern, complex economy goes through several stages of production and distribution, from acquisition of raw materials and intermediate goods, to manufacturing of the final good, to marketing and sales to final consumers. Each of these stages involves both firm-level (strategic) decisions and external factors that influence the final good outcome in terms of prices, quantities, quality, product differentiation and so on. Some of these activities take place inside the firm and others in markets. Economics, and economists, tend to focus on particular stages or interactions at the horizontal or vertical levels.

For example, there is a large literature, going back to Cournot (1838) and Bertrand (1883) that examines the strategic interactions of firms in a particular (horizontal) market. An important theme in this literature is how the choice of strategic variable (e.g., price versus quantity) impacts on market outcomes. More general representations of the strategy space are considered by Robson (1981), Klemperer and Meyer (1989), and Kao, Menezes and Quiggin (2014).

Similarly, there is a large and influential literature in industrial organization that focuses on how firms choose to organize their operations. Coase (1937) introduced the notion of transaction costs as an important factor in what is often called the make-or-buy choice. This refers to decisions about what should be done inside the firm, what should be contracted out and, if contracted, what should be the nature of contractual arrangements between the firm and outside parties.

Coase's ideas were extended by Williamson (1971) relating transaction costs to asset specificity and concerns about hold-up. Concerns about the possibility of particular assets being stranded lead naturally to a focus on the contracts that govern the relationship between the different parties. In this vein, Alchian and Demsetz (1972) proposed a theory of property rights and much later Hart (1995), Grossman and Hart (1986) and many others looked at contract incompleteness as one of the main drivers of the determinants of the nature of firms.[1]

This vast literature encompasses a wide range of vertical relationships from arms-length procurement to franchise contracts (including dealership arrangements) to full vertical integration.[2] A central theme is the notion that vertical integration harmonizes the interests of the different parties. For example, a manufacturer that sells directly to consumers avoids any potential conflicting incentives that an independent retailer may have. By contrast, if the retail market is competitive, and upstream firms lack information about the state of demand, they may find it difficult to exploit any market power they may have. Intermediate forms of organization such as dealerships may permit firms to gain some of the benefits of market power while separating production activities from the retail services required by final consumers.

[1] Demsetz (1988) himself made such a connection.

[2] For a comprehensive survey of the empirical literature on vertical integration and firms' boundaries, see Lafontaine and Slade (2007).

Our interest lies in the interface between the two central themes outlined above. Broadly speaking, we seek to better understand the relationship between the nature of competition in a final goods market and the firms' choices of vertical strategies.[3] More specifically, we focus on why firms with market power in a final goods market may choose specialist dealers, involving a franchise to sell specified items in a certain area, rather than exploiting their market power directly.

By and large, the literature on dealership contracts focuses on the case of a monopoly manufacturer. In this instance, a dealership contract, with resale price maintenance or exclusive territory clauses, eliminates double marginalization and the free rider problem that arises when sales require expenditure on promotional activities.[4] Instead, as in Bonanno and Vickers (1988), we focus on the case of oligopolistic manufacturers.

Whereas in Bonanno and Vickers (1988) manufacturers choose wholesale prices and retailers choose retail prices, in our model manufacturers make more complex choices. We develop a manufacturer/dealer model with second-stage oligopolistic competition in supply schedules with stochastic demand. Similar to Blair and Lewis (1994), the manufacturer does not know whether to attribute low sales to low demand or to the dealer skimping on services. Whereas Blair and Lewis investigate the properties of an optimal contract, we focus on costly monitoring by the manufacturer.

We assume that manufacturers give instructions to dealers who undertake the sales, upon observing the true state of demand. The dealer is paid a per unit service cost. Absent service, the good is worth less to the consumers, and thus the realized demand is lower for the firm. The manufacturer cannot distinguish between the lack of service and lower demand state without costly monitoring.

We model the dealership–manufacturer relationship by referring to a number of features that are present in existing dealership arrangements governing automobile distribution as documented by Arrunada, Garicano and Vasquez (2001). The reported contractual features include the manufacturer's right to determine (and monitor) sales targets and to set maximum authorized prices.

In our framework, manufacturers are assumed to set incentives that define a space of marketing strategies. These incentives determine the intensity of competition, represented by a willingness to discount their preferred price in order to achieve a target sales quantity. The polar cases are Bertrand strategies

[3] This is similar in spirit to the literature that seeks to understand how the nature of competition also seems to affect the internal organization (governance and delegation of tasks) inside the organisation. See the recent summary by Aghion, Bloom and Reenen (2013)

[4] See, for example, Telser (1960), Mattewson and Winter (1985) and Rey and Tirole (1986).

in which firms fix a price, and sell the quantity demanded at that price, and Cournot strategies in which firms fix a quantity and sell it at the market-clearing price. Between these two polar extremes are a range of possible marketing strategies, specified by a 'willingness to discount' parameter, β, ranging from zero (Cournot) to infinity (Bertrand).

In the case of a Bertrand strategy, the manufacturers' instructions are self-enforcing. The dealer is given a specified price. With price competition, absent service, the dealer is effectively selling the product at a higher price than the competitors, which leads to zero sales. Therefore the gain to the dealer from shirking service effort is zero. In this case, there is no need for costly monitoring of service provision. This strategy can also be thought of as a 'no-dealership' strategy; the dealer does not play an active role in the second-stage of the game.

In the Cournot case, by contrast, the dealer is given a sales target and instructed to sell at whatever the market clearing price. By shirking service effort and disguising the lower sales as a result of a lower realized demand state, the gain to the dealer is the saving on service cost. Hence, the firm must engage in costly monitoring to ensure that the dealer is providing adequate service as contracted and, thus, truthfully reporting the state of demand.

We show that if the monitoring cost parameter is to high, the firm might deprive the dealer of any decision power by behaving like a Bertrand competitor. For sufficiently low monitoring cost parameters, the firm chooses the degree of flexibility for its dealer. The equilibrium market outcome is indexed by the monitoring cost.

2 The Model

We model a two-stage, n firm oligopoly game where each firm is characterized by a pair comprising a manufacturer and a dealer. The inverse market demand is given by:

$$P(Q, \varepsilon, s) = F(Q, \varepsilon) + G(s),$$

where F is a continuous, concave function in its first argument, G a continuous, concave function of the service level s and $\varepsilon \in \mathbb{R}$ a stochastic shock with $E[\varepsilon] = 0$ and $Var[\varepsilon] = \sigma_\varepsilon^2$.

As the optimal level of service is a per-unit fixed amount, and since, as we will show that in equilibrium, the dealer will provide such an optimal amount, it is convenient to write the (direct) market demand as $D(P, \varepsilon)$ with $D_P < 0$, $D_{PP} \leq 0$.

In the first stage, each manufacturer i first chooses a 'permissible discount' rate (β_i) which specifies the degree of flexibility of the sales contract. As explained further below, this discount β_i is represented by the slope of the supply function chosen by the manufacturer and captures the intensity of competition. Then, given the choices $\beta_i, i = 1, ..., n$, each manufacturer chooses a price–quantity target pair $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$. The manufacturers can be thought of as choosing a pair (P_i, q_i) for each possible realization of ε. The stochastic shock ε is revealed at the second stage, and is observable only by the retailer. The contract requires the retailer to implement (P_i, q_i) according to the instructions of the manufacturer. However, since ε is unobservable to the manufacturer, the manufacturer can only audit the service provision through costly monitoring.

Manufacturer i can observe the quantity sold by its dealer, q_i, and by examining the invoices, the sales price P. The dealer is contracted to perform some service for each unit of sale and receives a fixed per unit amount. Since the demand state is unobservable to the manufacturer, the dealer can under-provide service and report lower ε. The gain to the dealer from service under-provision is the saving on per unit service cost.

To mitigate the ability and the incentives of its dealer to under-provide service and mis-report the demand state, manufacturer i incurs a monitoring cost $M_i(\bullet)$. The amount of monitoring cost required to facilitate the truthful reporting from the dealer depends on the latitude given to the dealer in terms of the permissible discount. As β increases, competition in the market intensifies. Without the service component provided, the dealer sells a smaller quantity, and thus there is less gain associated with shirking service. That is, the monitoring cost decreases in β.[5]

We will show that such a price–quantity target, together with the permissible discount variable β, gives rise to competition in linear supply functions. The slope of the supply curve is chosen in the first stage. In the second stage, the dealers play an oligopoly game in which the strategy space consists of the set of linear supply schedules with the given slope.

With competition in supply functions, given the strategies of other firms, the firm maximizes profit by choosing one point on its residual demand curve. Choosing quantity and choosing price give the same outcome. The observation of a Delta airlines executive, cited by Klemperer and Meyer (1989, footnote 5), that:

'We don't have to know if a balloon race in Albuquerque or a rodeo

[5] We present the monitoring game in the appendix.

in Lubbock is causing an increase in demand for a flight'

is apposite here. This observation remains true if the increased demand for Delta services is caused by a reduction in the number of flights offered by, say, Southwest.

Note, however, that the timing of the game differs from that of Klemperer and Meyer (1989), where firms must specify a supply schedule before learning the value of ε, and where it is assumed that the range of values of ε is sufficient to determine a complete supply schedule. In our case, the pair $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ depends on the reported state of demand.[6] Since firms are not price-takers, their desired price and quantity will, in general, increase with demand. The resulting locus of equilibrium prices and quantities has many of the properties of a supply curve, but must be interpreted differently. This point is developed further below.

In this setting, the strategy space for manufacturer i consists of vectors $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ where $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ is the 'target' price–quantity pair, and β_i is the willingness to discount. The vector $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ represents the instructions given by the firm, as the manufacturer, to its dealer.

Assuming constant marginal cost of production, the manufacturer's total costs are given by:

$$TC_i(\beta_i, q_i) = M_i(\beta_i) + cq_i + \delta q_i,$$

with $\beta_i \geq 0$, $q_i > 0$, $M_i^{'} < 0$, $M_i^{''} \geq 0$, $c \geq 0$ the marginal cost of production, and $\delta \geq 0$ the per unit sales service payment to the dealer.

Hence, for a given price–quantity pair (P, q_i), profit for manufacturer i, conditional on the choice of β_i and the demand shock ε, is given by:

$$\pi_i(q_i; \beta_i, \varepsilon) = Pq_i - TC_i(\beta_i, q_i).$$

We demonstrate that truth telling of the demand state by the dealer can be achieved with costly monitoring in Appendix A. Furthermore, the monitoring cost decreases in β. For $\beta = 0$, the firm wants the retailer to sell exactly the target output. This corresponds to Cournot competition with a fixed output, independent of the market price. When $\beta \to \infty$, for any $P \geq \widetilde{P}_i(\varepsilon)$, the firm is

[6] The manufacturer specifies $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ contingent on the true demand state. However, the true demand state is not observable to the manufacurer. The realised $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ depend on the reported demand state from the dealer. We show in the appendix that in equilibrium, the dealer reports the true demand state with sufficient costly monitoring.

willing to supply any quantity. This corresponds to Bertrand type behavior. In the Bertrand case, the dealers have no opportunity for cheating, since under-provision of service would lead to zero sales. The dealer in this case is best thought of as an employee of the manufacturer.

Given the instructions set by the manufacturers in the first stage, and following the realization of the demand shock ε, the dealers implement the strategies in the second period. In equilibrium, with all dealers truthfully reporting the demand state and provide the contracted service, a common price, $P(\varepsilon) = \widetilde{P}_i(\varepsilon)$, $\forall i$, emerges and firms sell their selected output $\widetilde{q}_i(\varepsilon)$, and the market clears with $D(P(\varepsilon), \varepsilon) = \sum_i \widetilde{q}_i(\varepsilon)$.

3 The main result

We solve the game backwards to obtain the subgame perfect Nash equilibrium. That is, we first derive the market equilibrium $(P(\varepsilon, \boldsymbol{\beta}), \mathbf{q}(\varepsilon, \boldsymbol{\beta}))$ where ε is the observed shock, $\boldsymbol{\beta} = (\beta_1, ..., \beta_n)$ is the vector of first round strategy choices and $\mathbf{q} = (q_1, ..., q_n)$ is the vector of equilibrium output quantities. We derive first-order conditions for the general case, and then a closed-form solution for the case where firms with zero marginal production cost compete in an industry with linear inverse demand curve, subject to an additive demand shock:

3.1 The choice of price and output target

In the product market competition stage, the dealer chooses the output and price given the instructions from the manufacturer. We show in the appendix that with costly monitoring, the dealer implements the output and price choice contingent on the true demand state as specified by the manufacture. We analyze the manufacturer's choice of the output and price target in this section given the first stage $\boldsymbol{\beta}$.

Recall that manufacturer i's instructions consist of a triple $\left(\beta_i, \widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ where $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$ represents the desired price–quantity pair for demand state ε, and $\beta_i \in [0, \infty]$ represents the willingness of firm i to discount the preferred price $\widetilde{P}_i(\varepsilon)$ in order to achieve the desired quantity $\widetilde{q}_i(\varepsilon)$. That is, for given ε, the firm instructs the retailer to offer a locus of price–quantity pairs (P, q) passing through $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$, such that when evaluated at $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$,

$$\frac{\partial \widetilde{P}_i(\varepsilon)}{\partial \widetilde{q}_i(\varepsilon)} = \frac{1}{\beta_i}. \tag{1}$$

Without loss of generality,[7] we can confine attention to the linear locus given by the equation

$$\beta_i \left(\hat{P} - \widetilde{P}_i (\varepsilon) \right) = (q_i - \widetilde{q}_i (\varepsilon)), \beta_i \in [0, \infty]$$

or

$$q_i \left(\hat{P}, \beta, \varepsilon \right) = \widetilde{q}_i (\varepsilon) + \beta_i \left(\hat{P} - \widetilde{P}_i (\varepsilon) \right), \beta_i \in [0, \infty] \tag{2}$$

where $q_i \left(\hat{P}; \beta, \varepsilon \right)$ is the quantity sold by the dealer and $\hat{P}$ is the market clearing price with $D \left(\hat{P}; \varepsilon \right) = \sum_{i=1}^{n} q_i \left(\hat{P}; \beta_i, \varepsilon \right)$. The case $\beta_i = 0$ $\forall i$ gives a family of vertical supply schedules. This corresponds to Cournot competition with a fixed output.

The other polar case when $\beta = \infty$ yields horizontal supply schedules. More precisely:

$$q_i \left(\widetilde{P}_i, \hat{P}, \beta, \varepsilon \right) = \begin{cases} D \left(\widetilde{P}_i; \varepsilon \right) & \hat{P} > \widetilde{P}_i (\varepsilon), \\ [0, D \left(\widetilde{P}_i; \varepsilon \right)] & \hat{P} = \widetilde{P}_i (\varepsilon), \\ 0 & \hat{P} < \widetilde{P}_i (\varepsilon), \end{cases}$$

That is, the firm will supply the entire market at prices above the target $\widetilde{P}_i (\varepsilon)$, zero at prices below $\widetilde{P}_i (\varepsilon)$, and any output in the range $[0, D \left(\widetilde{P}_i; \varepsilon \right)]$ when $\hat{P} = \widetilde{P}_i (\varepsilon)$. This corresponds to Bertrand type behavior. The choice of $\beta = \infty$ permits any deviation from the target output and therefore does not incur any monitoring cost.

For known ε and β_j, and given choices of $\left(\widetilde{P}_j (\varepsilon), \widetilde{q}_j (\varepsilon) \right)$,$\forall j \neq i$, each firm takes its residual demand curve as given, and acts as a profit-maximizing monopolist.[8]

Market clearing gives

$$D \left(\hat{P}; \varepsilon \right) = \sum_{j=1}^{N} q_j \left(\hat{P}; \beta_i, \varepsilon \right) = q_i \left(\hat{P}; \beta_i, \varepsilon \right) + Q_{-i} \left(\hat{P}; \beta, \varepsilon \right)$$

where

$$Q_{-i} \left(\hat{P}; \beta, \varepsilon \right) = \sum_{j \neq i} q_j \left(\hat{P}; \beta_{-i}, \varepsilon \right).$$

[7] As shown by Kao, Menezes and Quiggin (2014), if $\left(\widetilde{P} (\varepsilon), \widetilde{q}_i (\varepsilon) \right)$, $i = 1, ..., n$, is an equilibrium loci of the form (2), conditional on demand $D(P, \varepsilon)$, it is an equilibrium for any game in which, for all i, the strategy space is given by loci $\hat{q}_i (P, \beta, \varepsilon)$ satisfying (1) for each $\left(\widetilde{P}_i (\varepsilon), \widetilde{q}_i (\varepsilon) \right)$. All such strategies may be summarized by the pair $(\beta_i, \widetilde{q}_i (\varepsilon))$ such that (2) holds in a neighborhood of $\widetilde{P}_i$.

[8] Choosing $\left(\widetilde{P}_i (\varepsilon), \widetilde{q}_i (\varepsilon) \right)$, combined with the constraint $\hat{P} = \widetilde{P}_i (\varepsilon)$ in equilibrium, gives us the same solution as choosing the supply schedule $q_i \left(\hat{P}; \beta, \varepsilon \right)$.

The residual demand facing firm i, represented by its dealer, in the second stage is

$$D_i\left(\hat{P};\varepsilon\right)=D\left(\hat{P};\varepsilon\right)-Q_{-i}\left(\hat{P};\beta,\varepsilon\right).$$

Firm i solves

$$\max_{q_i}\pi_i\left(q_i;\beta_i,\varepsilon\right)=D_i\left(\hat{P};\varepsilon\right)\left(\hat{P}-c-\delta\right)-M_i\left(\beta_i\right).$$

The FOC gives

$$\left(\hat{P}-c-\delta\right)\left(D'\left(\hat{P};\varepsilon\right)-Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\frac{\partial\hat{P}}{\partial q_i}+D_i\left(\hat{P};\varepsilon\right)\frac{\partial\hat{P}}{\partial q_i}\leq 0.$$

with equality for interior solutions.

In an interior solution, the optimal q_i^* in the second stage is defined by

$$\left(\hat{P}-c-\delta\right)\left(D'\left(\hat{P};\varepsilon\right)-Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)+D_i\left(\hat{P};\varepsilon\right)=0. \tag{3}$$

Note that this is just the familiar inverse elasticity pricing rule with the residual demand facing firm i adjusted by the supply function. At $\hat{P}$, the residual demand elasticity, ξ, is

$$\xi=\frac{\frac{\partial q_i}{q_i^*}}{\frac{\partial\hat{P}}{\hat{P}}}=\frac{\partial q_i}{\partial\hat{P}}\frac{\hat{P}}{q_i^*}=\left(D'\left(\hat{P};\varepsilon\right)-Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\frac{\hat{P}}{q_i^*}.$$

Simple re-arrangement of Equation 3 gives

$$\frac{\hat{P}-c-\delta}{\hat{P}}=-\frac{q_i^*}{\left(D'\left(\hat{P}\right)-Q'_{-i}\left(\hat{P};\beta,\varepsilon\right)\right)\hat{P}}=-\frac{1}{\xi}. \tag{4}$$

Remark 1 *Firms' second stage quantity choices are strategic substitutes.*

3.2 First-stage equilibrium

We now consider the determination of β in the first stage of the game.

In the first stage, firm i chooses β_i to maximize

$$\max_{\beta_i}E\left[q_i^*\left(\beta_i\right)\hat{P}\left(q_i^*\left(\beta_i\right),Q_{-i}^*\right)-TC_i\left(\beta_i,q_i^*\right)\right].$$

The FOC yields:

$$E\left[\frac{\partial q_i^*\left(\beta_i\right)}{\partial\beta_i}\left(\hat{P}\left(q_i^*\left(\beta_i\right),q_{-i}^*\right)-c-\delta\right)+q_i^*\frac{\partial\hat{P}\left(q_i^*\left(\beta_i\right),Q_{-i}^*\right)}{\partial\beta_i}-\frac{\partial M_i\left(\beta_i\right)}{\partial\beta_i}\right]\begin{array}{c}<\\=\\>\end{array}0, \tag{5}$$

with $<$ for $\beta_i^* = 0$, $=$ for $\beta_i^* \in (0, \infty)$, and $>$ for $\beta_i^* = \infty$.

We have

Proposition 1 $\beta_i = \infty, \forall i$ *is a first-stage equilibrium, with the corresponding second-stage solution* $\hat{P}(\varepsilon) = c + \delta$, $Q(\varepsilon) \equiv D(c + \delta, \varepsilon)$.

Proof. Given $\beta_j = \infty$, $\forall j \neq i$, from Equation 4, we have $\hat{P} = c + \delta$ and hence $\frac{\partial \hat{P}}{\partial \beta_i} = 0$. With $\frac{\partial M_i(\beta_i)}{\partial \beta_i} < 0$, $\beta_i^* = \infty$. ■

Proposition 1 establishes that there is always an equilibrium where the dealer plays no active role in the product market. In essence, we have shown that there is always a no-dealership equilibrium under imperfect competition. This result offers, for example, some insights into the changing role of car dealerships upon the introduction of strong competition from imports and from the increase in the intensity of competition that has been brought about by the internet. For example, recent analysis suggests that

> *'the average number of showroom visits a customer makes before a new car purchase is on an average 1.4 – down from four visits. Customers walk into a dealership with their homework done. They will have browsed websites, read reviews, visited social networks and community forums – and at that point, the role of the dealer will no longer be that of an information source, but that of a product experience provider.'*[9]

Proposition 1 raises the question of whether it is possible to have an equilibrium where dealers play a more active role under imperfect competition. Below we answer this question in the affirmative and in the next subsection we provide sufficient conditions for such an equilibrium to exist.

Proposition 2 *For some range of marginal monitoring cost, there exists a positive profit symmetric equilibrium.*

Proof. As $\frac{\partial TC_i(\beta_i, q_i)}{\partial q_i} = c + \delta, \forall i$, in an interior solution, the FOC in Equation 5 gives

$$E\left[\frac{\partial q_i^*(\beta_i)}{\partial \beta_i}\left(\hat{P}\left(q_i^*(\beta_i), q_{-i}^*\right) - c - \delta\right) + q_i^* \frac{\partial \hat{P}\left(q_i^*(\beta_i), q_{-i}^*\right)}{\partial \beta_i}\right] = E\left[\frac{\partial M_i(\beta_i)}{\partial \beta_i}\right]. \tag{6}$$

As shown in the proof of Proposition 1, the LHS is equal to 0 when $\beta = \infty$.

[9] See *http://www.forbes.com/sites/sarwantsingh/2014/02/05/the-future-of-car-retailing/*

At $\beta = 0$, the LHS becomes

$$E\left[\frac{-2D'\left(\hat{P}\right)(n-1)\left(\hat{P}-c-\delta\right)^2}{(1+n)\,D'\left(\hat{P}\right)+nD''\left(\hat{P}\right)\left(\hat{P}-c-\delta\right)}\right] < 0.$$

Given that the LHS is continuous in β, for $E\left[\frac{\partial M_i(\beta_i)}{\partial \beta_i}\right] \in \left(E\left[\frac{-2D'\left(\hat{P}\right)(n-1)\left(\hat{P}-c-\delta\right)^2}{\left((1+n)D'\left(\hat{P}\right)+nD''\left(\hat{P}\right)\left(\hat{P}-c-\delta\right)\right)}\right], 0\right)$, there exists a symmetric positive profit equilibrium such that $\beta^* \in (0, \infty)$, and the FOC at β^* is satisfied for all firm i. ■

3.3 Dealership equilibrium with linear demand and additive shocks

The aim of this subsection is to uncover some conditions that are associated with the emergence of dealership equilibria – equilibria where dealers play an active role in setting prices to adjust to demand conditions. To this end, we will focus on the simple case where n firms compete in an industry with a linear inverse demand curve subject to an additive demand shock while both c and δ are normalized to zero. Let the inverse demand be

$$P = 1 - b\sum_{i=1}^{n} q_i + \varepsilon. \tag{7}$$

We show in the appendix that the monitoring cost is decreasing in β and is equal to 0 when $\beta = \infty$. For this example, we assume that the monitoring cost is given by $M_i = \frac{\theta}{\beta_i}$, where θ is a common parameter for all firms. With these simplifications, we can derive the closed form solution to the second-stage game and characterize the first-stage choices of β as follows:

Proposition 3 *For $0 < \theta < \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}$, there exists, in addition to the Bertrand equilibrium, a unique symmetric equilibrium $\beta = \beta(\theta) \in (0, \infty)$, with second-stage equilibrium*

$$q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b}+(n-1)\beta\right)}{n+1+b(n-1)n\beta}, Q = \frac{n(1+\varepsilon)\left(\frac{1}{b}+(n-1)\beta\right)}{n+1+b(n-1)n\beta}, P = \frac{(1+\varepsilon)}{n+1+b(n-1)n\beta}.$$

In equilibrium,

$$\pi^* = \frac{\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{b}+(n-1)\beta\right)}{\left(n+1+b(n-1)n\beta\right)^2} - \frac{\theta}{\beta}.$$

Given the FOC (Equation 12)on β and the implied relationship between β and θ, $\pi^* > 0$. Further, this equilibrium is stable.

The proof of Proposition (3) is in the appendix. The following corollary summarizes the sufficient conditions for the existence of a positive-profit, dealership equilibrium.

Corollary 1 *For linear demand, and monitoring costs given by $M_i = \frac{\theta}{\beta_i}$, a positive-profit equilibrium where dealers play an active role in setting prices is more likely whenever the number of competitors is small, the variance of demand is high, and the the demand function is steeper.*

The potential multiplicity of equilibria introduces the familiar question about equilibrium selection. While in general we do not know which equilibrium is stable, Proposition (3) establishes that the positive profit equilibrium is stable. For the special case of linear demand and $M_i = \frac{\theta}{\beta_i}$, we can show that the Bertrand equilibrium is not stable. To see this, we may observe that firms' first-stage β choices are strategic complements when all firms' choices of β are close to each other. For any given $j \neq i$:

$$\frac{\partial^2 E\pi_i}{\partial\beta_i\partial\beta_j} = -\frac{2\left(1+\sigma_\varepsilon^2\right)b\left(n-1\right)}{\left(1+n+b\left(n-1\right)\sum_{i=1}^N\beta_i\right)^3}+\frac{6\left(1+\sigma_\varepsilon^2\right)b\left(1+b\sum_{j\neq i}^N\beta_j\right)\left(n-1\right)^2}{\left(1+n+b\left(n-1\right)\sum_{i=1}^N\beta_i\right)^4}, \tag{8}$$

with $\frac{\partial^2 E\pi_i}{\partial\beta_i\partial\beta_j} > 0$ for close enough β_i and β_j and $n > 1$. This implies that in this case, the Bertrand equilibrium is unstable. To see that this is not generally true, note that if the monitoring cost includes a fixed component that is not incurred for the case when $\beta \to \infty$, then the Bertrand equilibrium is stable.

The polar cases of the second-stage solution span the range of (weakly positively sloped) supply-schedule equilibria. When $\beta = \infty$, $P = \pi_i = 0$, $q_i = \frac{1+\varepsilon}{bn}$, $Q = \frac{1+\varepsilon}{b}$. This is the Bertrand solution. For $\beta = 0$, $P = \frac{1+\varepsilon}{n+1}$, $q_i = \frac{1+\varepsilon}{b(n+1)}$, $Q = \frac{n(1+\varepsilon)}{b(n+1)}$ and the gross profit $\pi_i + \frac{\theta}{\beta} = \frac{1+\sigma_\varepsilon^2}{b(n+1)^2}$. This is the Cournot solution.

To show that these polar cases arise as equilibria for the full game, we require

Proposition 4 *As $\theta \to \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2n^3(n-1)}$ from below, $\beta \to \infty$, and the equilibrium outcome converges to the Bertrand solution. As $\theta \to 0$, $\beta \to 0$ and the equilibrium outcome converges to the Cournot solution. More generally, For $0 < \theta < \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2n^3(n-1)}$, as θ increases, the interior symmetric equilibrium β increases.*

In general, in oligopoly problems, consumer surplus and producer surplus move in opposite directions. However, welfare analysis in this model is complicated by the cost $\frac{\theta}{\beta}$ incurred by firms in the first stage. Expected consumer surplus in a interior symmetric equilibrium is

$$ECS = E\left[\frac{1}{2}\left(1+\varepsilon-P\right)Q\right] = \frac{b\left(1+\sigma_\varepsilon^2\right)}{2}\left(\frac{n\left(\frac{1}{b}+(n-1)\beta\right)}{n+1+b\left(n-1\right)n\beta}\right)^2.$$

The expected total surplus is defined to be the sum of firms' expected profits and expected consumer surplus. Thus

$$ETS = n\left(\frac{\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{b}+(n-1)\beta\right)}{\left(n+1+b\left(n-1\right)n\beta\right)^2}-\frac{\theta}{\beta}\right)+\frac{b\left(1+\sigma_\varepsilon^2\right)}{2}\left(\frac{n\left(\frac{1}{b}+(n-1)\beta\right)}{n+1+b\left(n-1\right)n\beta}\right)^2.$$

We now present some numerical examples.

Example 1 *For $b=1$, $n=2$, $\theta=0.03$, $\sigma_\varepsilon^2=0.01$ the symmetric equilibrium gives $\beta\approx 0.952$, $Eq\approx 0.398$, $EP\approx 0.20, E\pi\approx 5.047\times 10^{-2}$, and $ETS\approx 0.421$. For $b=1$, $n=2$, $\theta=0.1$ and , $\sigma_\varepsilon^2=0.01$, the symmetric equilibrium gives $\beta\approx 3.050$, $Eq\approx 0.445$, $EP\approx 0.11$, $E\pi\approx 1.661\times 10^{-2}$, and $ETS=0.433$.*

We further explore the relationships between the optimal symmetric β, the firm's profit, the market price, the the total surplus and θ in the diagram below. The diagram below is plotted with $\sigma_\varepsilon^2=0.01$.

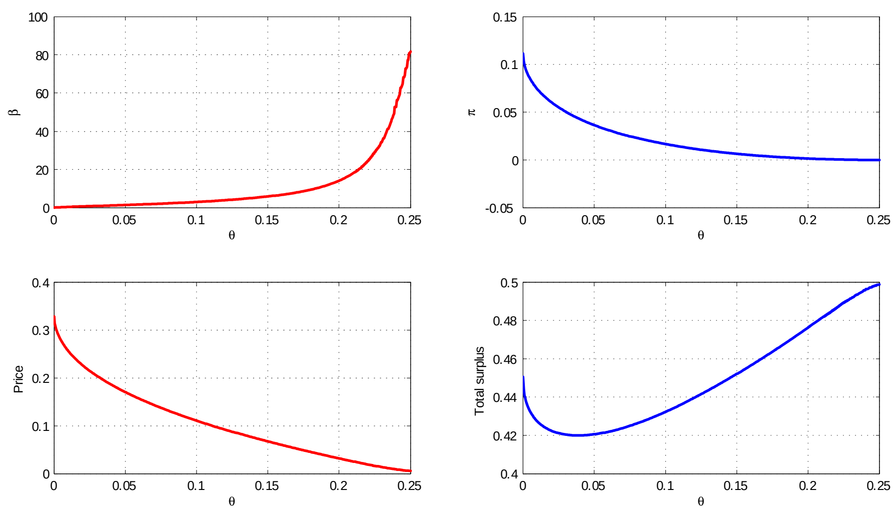

Figure 1: Symmetric Equilibrium β, π, EP, ETS. Plotted with $b=1$, $n=2$, $\sigma_\varepsilon^2=0.01$.

4 Conclusion

Little is known about the interaction between the nature of competition in output markets and the different vertical arrangements that prevail in a modern economy, varying from arms-length transactions to contractual arrangements such as dealerships to full vertical integration. This paper is a first attempt at examining whether dealership arrangements can emerge in equilibrium when manufacturers compete in the output market.

Generally, we show that there is always an equilibrium where dealers play no active role in setting prices in the output market. We refer to this as the non-dealership equilibrium. This equilibrium is characterized by intense competition, approaching the limiting case of Bertrand. We also show that under some circumstances, such as a small number of competitors, or inelastic or more volatile demand, another equilibrium exists where dealers are active at price setting. Our results seem to fit with existing trends in dealership arrangements for car companies, where dealers become providers of product experience rather than sales services.

5 Appendix A: The Monitoring Game

We uses a linear demand example to illustrate the monitoring game between the manufacturer and its dealer.[10] We show that with a properly specified penalty payment, the dealer truthfully reports the demand state and carries out the specified price and output target $\left(\widetilde{P}_i(\varepsilon), \widetilde{q}_i(\varepsilon)\right)$. The dealer in our model is paid a per unit service payment δ. Due to demand uncertainty, the dealer can potentially choose not to provide service and attribute low demand to lower realization of the random shock. We view the goods being sold as a product with the service component included, $D(P, s, \varepsilon)$, where $D_1 < 0$, $D_{11} \leq 0$, $D_2 > 0$, s is the service provided by the retailer, and $\varepsilon \in \mathbb{R}$ a stochastic shock with $E[\varepsilon] = 0$ and $Var[\varepsilon] = \sigma_\varepsilon^2$.

Consumers' valuation for the product, net of the service component is

$$P - G(s).$$

[10] The example presented is a two firm analysis. However, we analyse the potential deviation of one dealer while all other dealers are playing the equilibrium strategies. The price charged by the other dealer can be interpreted as the price charged by all other dealers, and the quanitity sold by other dealers can be interpreted as the aggregate quantity of all of all other dealers. The example can be extended and results apply to n firm analysis.

where $G(s)$ is the value added from the service component with $G'(s) > 0$ and $G''(s) < 0$. With two firms, in equilibrium, we have

$$P_1 - G_1(s_1) = P_2 - G_2(s_2).$$

Let s^* denote the optimal service level per unit of sale from the manufacturer's point of view and e^* the resulting premium with optimal service level, $G(s^*) = e^*$. The cost to the dealer for performing s^* amount of service is δ. The dealer gets zero profit by exerting the contracted service for each unit of output sold. Given that dealer 2 is offering (q_2, s^*), the demand facing dealer 1 is determined:

$$P_1 - G_1(s_1) = P_2 - e^* = 1 + \varepsilon - q_1 - q_2 - e^*.$$

Thus, if the dealer under-provides service with $s_1 < s^*$, the sticker price is reduced, $P_1 < P_2$.

5.1 Truth Telling equilibrium

If dealer 1 exerts effort $s_1 = s^*$ with $G_1(s_1^*) = e^*$, we have the market clearing condition with

$$P_1 = P_2 = 1 + \varepsilon - q_1 - q_2.$$

Both firms produce with marginal costs $c + \delta$. Firms compete in supply schedules with $\frac{\partial q_i}{\partial P} = \beta_i$. Given (q_2, β_2)

$$P = 1 + \varepsilon - q_1(P, \beta_1) - q_2(P, \beta_2).$$

The FOC for q_i determination satisfies

$$\frac{(-1 - \beta_2)}{-1 - \beta_1 - \beta_2}(1 + \varepsilon - q_1 - q_2 - c - \delta) + \frac{q_1^*}{-1 - \beta_1 - \beta_2} = 0.$$

The equilibrium quantities are

$$q_1 = \frac{(1 + \varepsilon - c - \delta)(1 + \beta_2)}{\beta_1 + \beta_2 + 3} \text{ and } q_2 = \frac{(1 + \varepsilon - c - \delta)(1 + \beta_1)}{\beta_1 + \beta_2 + 3}.$$

In this truth telling equilibrium, the dealer incurs a δ per unit cost for service and provides service s^*. The dealer is compensated for exactly δ per unit sold and earns zero profit.

5.2 Deviation

Now consider a possible deviation. Given firm/dealer 2's strategy (q_2, s^*), if dealer 1 chooses to deviate and provide $s_1 = 0$,

$$P_1 = P_2 - e^* = 1 + \varepsilon - q_1 - q_2 - e^*.$$

Firm 1 charges a lower sticker price than firm 2 to reflect lower willingness to pay from the consumer for the product net of the service component. Given q_2, we have

$$P_1 = 1 + \varepsilon - q_1 - \frac{(1 + \varepsilon - c - \delta)(1 + \beta_1)}{\beta_1 + \beta_2 + 3} - e^*.$$

To report a ε' consistent with (q_1, s^*), the dealer reports

$$\varepsilon' = \varepsilon - e^*.$$

Given this, the resulting instruction from the manufacturer would specify

$$q_1(\varepsilon') = \frac{(1 + \varepsilon - e^* - c - \delta)(1 + \beta_2)}{\beta_1 + \beta_2 + 3}.$$

The gain to the dealer is the the per unit gain of service cost,

$$S = \frac{(1 + \varepsilon - e^* - c - \delta)(1 + \beta_2)}{\beta_1 + \beta_2 + 3}\delta. \tag{9}$$

S is decreasing in β_1 and $\to 0$ as $\beta_1 \to \infty$.

Lemma 1 *Suppose that the manufacturer monitors the service report with probability $\psi > \frac{S}{\Delta}$, where Δ is a fixed penalty if misreporting is observed, and that monitoring costs are affine in ψ and the monitoring costs M_i are proportional to S:*

$$M_i = m_i + S_i,$$

where m_i is the fixed cost for monitoring. Then the dealer reports the true state of demand.

Proof. By inspection of (9). ■

Observe in particular that, if $m_i = 0$, in the case of the Bertrand strategy space $\beta_i \to \infty$, we have $M_i \to 0$.

6 Appendix B

Proof of Proposition (3). Assuming zero production costs, the first-order condition 3 becomes

$$\left(\frac{1}{b} + \sum_{j \neq i} \beta_j\right)(1 - bQ + \varepsilon) - q_i = 0 \tag{10}$$

yielding the best response

$$q_i = \frac{\left(\frac{1}{b} + \sum_{j\neq i}\beta_j\right)(1 - bQ_{-i} + \varepsilon)}{\left(2 + b\sum_{j\neq i}\beta_j\right)}.$$

From FOC (10), we have

$$q_i = \left(\frac{1}{b} + \sum_{j\neq i}^{N}\beta_j\right)(1 + \varepsilon - bQ).$$

Summing up the n, $n = 1, ..., n$, FOCs we have

$$Q = \left(\frac{n}{b} + (n-1)\sum_{i=1}^{N}\beta_i\right)(1 - bQ + \varepsilon).$$

This gives

$$Q = \frac{(1+\varepsilon)\left(\frac{n}{b} + (n-1)\sum_{i=1}^{N}\beta_i\right)}{\left(1 + n + b(n-1)\sum_{i=1}^{N}\beta_i\right)},$$

$$P = \frac{(1+\varepsilon)}{n + 1 + b(n-1)\sum_{i=1}^{N}\beta_i}, \text{ and } q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b} + \sum_{j\neq i}^{N}\beta_j\right)}{1 + n + b(n-1)\sum_{i=1}^{N}\beta_i}.$$

■

Remark 2 *In a symmetric equilibrium, we have* $\beta_1 = \beta_2 = ... = \beta_n = \beta$ *and*

$$q_i^* = \frac{(1+\varepsilon)\left(\frac{1}{b} + (n-1)\beta\right)}{1 + n + b(n-1)n\beta},$$

$$Q = \frac{(1+\varepsilon)n\left(\frac{1}{b} + (n-1)\beta\right)}{1 + n + b(n-1)n\beta}, \text{ and } P = \frac{(1+\varepsilon)}{1 + n + b(n-1)n\beta}.$$

We now consider the choice of β_i in the first stage. Given the second stage outcome, firm i solves

$$\max_{\beta_i} E\left[\pi_i\left(\beta_i, \boldsymbol{\beta}_{-i}, \varepsilon\right)\right] = \left(\frac{(1+\varepsilon)\left(\frac{1}{b} + \sum_{j\neq i}^{N}\beta_j\right)}{1 + n + b(n-1)\sum_{i=1}^{N}\beta_i}\right)\left(\frac{(1+\varepsilon)}{n + 1 + b(n-1)\sum_{i=1}^{N}\beta_i}\right) - \frac{\theta}{\beta_i}.$$

The FOC gives

$$\frac{\partial E\left[\pi_i\left(\beta_i, \boldsymbol{\beta}_{-i}, \varepsilon\right)\right]}{\partial \beta_i} = -\frac{2\left(1+\sigma_\varepsilon^2\right)\left(1 + b\sum_{j\neq i}^{N}\beta_j\right)(n-1)}{\left(1 + n + b(n-1)\sum_{i=1}^{N}\beta_i\right)^3} + \frac{\theta}{\beta_i^2} \leq 0 \quad (11)$$

with equality for an interior solution.

In a symmetric interior solution

$$\theta = \frac{2\left(1+\sigma_\varepsilon^2\right)\left(1+b\left(n-1\right)\beta\right)\left(n-1\right)\beta^2}{\left(1+n+b\left(n-1\right)n\beta\right)^3}. \tag{12}$$

The RHS is equal to 0 if $\beta = 0$ and is equal to $\frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}$ if $\beta = \infty$. Furthermore, the RHS increases in β for $n \geq 1$. Given this monotonicity, for each given $\theta \in \left(0, \frac{2\left(\sigma_\varepsilon^2+1\right)}{b^2 n^3 (n-1)}\right)$, there is a unique symmetric β solution.

Remark 3 *The second order condition is always satisfied for symmetric equilibria with finite positive β.*

Substitute the θ value for an interior β solution from Equation 11,

$$\frac{\partial^2 E\pi_i\left[\pi_i\left(\beta_i, \boldsymbol{\beta}_{-i}^*\right)\right]}{\partial \beta_i^2} = 2\left(1+\sigma_\varepsilon^2\right)\left(n-1\right)\left(1+b\sum_{j\neq i}^{N}\beta_j\right)\left(\frac{b\left(n-1\right)\beta_i - 2\left(1+n\right) - 2b\left(n-1\right)\sum_{j\neq i}^{N}\beta_i}{\beta_i\left(1+n+b\left(n-1\right)\sum_{i=1}^{N}\beta_i\right)^4}\right). \tag{13}$$

$\frac{\partial^2 E\pi_i\left[\pi_i\left(\beta_i, \beta_{-i}, \varepsilon\right)\right]}{\partial \beta_i^2} < 0$ *for close enough β_i and β_{-i}.*

Proof. (continued) Proof of Stability

This proof follows Dixit (1986), Martin (2002), and Seade (1980), adapted to our setup. Let $\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)$ be an equilibrium of the first stage game. Suppose that if firms choose $\left(\beta_i, \boldsymbol{\beta}_{-i}^*\right)$ in the neighborhood of $\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)$, firm i changes β_i over time at a rate proportional to its marginal profitability,

$$\frac{d\beta_i}{dt} = k_i \frac{\partial E\pi_i\left(\beta_i, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i}, \tag{14}$$

for $k_i > 0$. That is, if it is profitable to increase β_i, firm i increases β_i at a rate which is proportional to marginal profitability.

Take a local linear approximation to Equation 14 around $\left(\beta_i^*, \beta_{-i}^*\right)$:

$$\frac{d\beta_i}{dt} = k_i \frac{\partial E\pi_i\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i} + k_i\left[\frac{\partial^2 E\pi_i\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i^2}\left(\beta_i - \beta_i^*\right) + \sum_{j\neq i}\frac{\partial^2 E\pi_i\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i \partial \beta_j}\left(\beta_{-i} - \beta_j^*\right)\right].$$

At an interior $\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)$, $\frac{\partial E\pi_i\left(\beta_i, \boldsymbol{\beta}_{-i}^*\right)}{\partial \beta_i} = 0$. Repeat the analysis for each firm

j, $j \neq i$, the system of adjustment equations can be written as

$$\begin{pmatrix} \frac{d\beta_1}{dt} \\ \frac{d\beta_2}{dt} \\ . \\ . \\ . \\ \frac{d\beta_n}{dt} \end{pmatrix} = \begin{pmatrix} k_1 & 0...0 & 0 \\ & k_2 & \\ 0 & . & 0 \\ & & . \\ 0 & 0...0 & k_n \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial^2 E\pi_1(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_1^2} & \frac{\partial^2 E\pi_1(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_1 \partial \beta_2} ... & \frac{\partial^2 E\pi_1(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_1 \partial \beta_n} \\ \frac{\partial^2 E\pi_2(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_2 \partial \beta_1} & \frac{\partial^2 E\pi_2(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_2^2} ... & \frac{\partial^2 E\pi_2(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_2 \partial \beta_n} \\ . & . & . \\ . & . & . \\ . & . & . \\ \frac{\partial^2 E\pi_n(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_n \partial \beta_1} & \frac{\partial^2 E\pi_n(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_n \partial \beta_2} ... & \frac{\partial^2 E\pi_n(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_n^2} \end{pmatrix} \begin{pmatrix} \beta_1 - \beta_1^* \\ \beta_2 - \beta_2^* \\ . \\ . \\ . \\ \beta_n - \beta_n^* \end{pmatrix}.$$

Note that the matrix on the LHS is a $n \times 1$ matrix. The matrices on the RHS are $n \times n$, $n \times n$, and $n \times 1$ respectively. Stability requires the Jacobian matrix to have a negative trace and that the determinant of the Jacobian matrix should have the same sign as $(-1)^n$. Given that the second order condition is satisfied in equilibrium, the Jacobian matrix has a negative trace. The determinant of the matrix can be computed as (see Dixit, 1986, and Seade, 1980):

$$\prod_{i=1, j\neq 1}^{n} \left(\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j} \right) \left(1 + \sum_{i=1, j\neq i}^{n} \frac{\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j}}{\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j}} \right).$$

The second condition we require is thus

$$(-1)^n \left[\prod_{i=1, j\neq 1}^{n} \left(\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j} \right) \left(1 + \sum_{i=1, j\neq i}^{n} \frac{\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j}}{\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} - \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j}} \right) \right] > 0. \tag{15}$$

The simplest set of sufficient conditions is obtained by requiring diagonal dominance in the matrix:

$$\left| \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} \right| > (n-1) \left| \frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i \partial \beta_j} \right|. \tag{16}$$

With symmetry, Equations 13 and 8 become

$$\frac{\partial^2 E\pi_i(\beta_i^*, \boldsymbol{\beta}_{-i}^*)}{\partial \beta_i^2} = 2\left(1+\sigma_\varepsilon^2\right)(n-1)(1+b(n-1)\beta) \left(\frac{b(n-1)\beta - 2(1+n) - 2b(n-1)^2 \beta}{\beta(1+n+b(n-1)n\beta)^4} \right).$$

$$\frac{\partial^2 E\pi_i\left(\beta_i^*, \boldsymbol{\beta}_{-i}^*\right)}{\partial\beta_i\partial\beta_j} = -\frac{2\left(1+\sigma_\varepsilon^2\right)b\,(n-1)}{\left(1+n+b\,(n-1)\,n\beta\right)^3}+\frac{6\left(1+\sigma_\varepsilon^2\right)b\left(1+b\,(n-1)\,\beta\right)(n-1)^2}{\left(1+n+b\,(n-1)\,n\beta\right)^4}.$$

Condition 16 is satisfied for $n > 1$. ■

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