Consumption rights: a market mechanism to redistribute wealth*

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Abstract

In an exchange economy with only private consumption goods we propose a competitive mechanism to reach any income distribution. We introduce the so called consumption rights, which is a real parameter that modifies the budgetary constraint of individuals but does not participate in the utility functions. Consumption rights can be traded in the market, which is the main difference with slack parameters, as fiat money or tax inflation, widely known as methods to modify the distribution of wealth. The policy maker control variables are both the amount of rights assigned to each individual and a pricing rule that defines the rate of exchange between rights and wealth. The only intervention of the planner will be trough the definition of the policy, because the redistribution of wealth will be the consequence of the competitive exchange among consumers.

Keywords: income redistribution, competitive equilibrium, consumption rights.

JEL Classification: D31, D51, D63, H21.

1 Introduction

Since Arrow-Debrew's existence of equilibrium theorem (see [1] and [10]), a great deal of effort has been made to generalize the hypotheses required to prove this classical result (see [8] as a general reference). The equilibrium allocation satisfies a set of desirable properties, including Pareto optimality, belonging to the core of the economy, and individual rationality among others[1]. Thus, a fundamental conclusion is that under very general hypotheses on the economy, the "invisible hand" leads to a solution that

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[1]See [13] as a general reference on the equilibria properties and for more general economic frameworks, for instance with increasing returns in production and/or infinite many agents, see [7] and references there in.

guarantees all these properties, and is regarded as an efficient outcome without the need of intervention by a policy maker.

Nevertheless, despite the merits of the competitive allocation, the view that the market is capable of assigning resources appropriately is not generally shared by most economists, mainly because the outcome does not necessarily fulfill a complementary set of consensus on equity criteria. Clearly, the vast literature on social choice supports the need for some complementary requirements.

In fact, economic theory has developed different methods to obtain efficient but also equitable outcomes. Among these methods, a lump-sum transfer of initial endowments has the advantage of avoiding undesired distortions in the assignment, thus maintaining efficiency in the economy that justifies why it is preferred to other tax systems.

However, in practice there are serious and well established difficulties with the implementation of lump-sum taxes. First, tax collection is far from costless, as was shown in the poll tax of the UK (see [21], p. 46), which reduces their efficient performance. Secondly, and more fundamentally, optimal lump-sums depend on all the relevant variables in the economy, many of them only known by individuals and not directly observable by the government, which ultimately relies on reported information. Deceptively, Mirrlees (see [17]) presents theorems that prove the impossibility of designing non-manipulable lump-sum taxes, which makes it ultimately impossible to define optimal lump-sums. Confronting such implementation difficulties we propose an alternative method to re-distribute endowments.

In this paper we consider an exchange economy, with a finite number of consumers and goods, without public goods or externalities. We extend the Arrow-Debreu model (see [1] and [10]) introducing a parameter that modifies the budgetary set but does not affect consumer preferences. We decided to interpret this parameter as tradable "consumption rights", although another plausible interpretation is as a parallel currency. In this new economy, called r-economy, the initial endowment of consumers consists not only of commodities (as usual), but of a number of rights centrally distributed to individuals at no cost to them; however, these rights have no role in the utility function. Every transaction of commodities implies two payments: a price in wealth (as usual) and a certain amount of rights. These rights are tradable at a price and are generic, i.e. they are valid to purchase any commodity.

An existence of equilibrium theorem in the r-economy is proved under usual assumptions on the economy. We also prove that, at equilibrium, tradable rights induce a redistribution of the initial endowments among consumers.

With these results, the exchange process with consumption rights has similar merits as the lump-sum method to reassign resources in the economy: both of them avoid price distortions and decentralize desirable Pareto optima. However, a significant difference between these methods is that by exchanging consumption rights, the wealth is redistributed without the intervention of a policy maker that collects and assigns resources in the economy. The only role of the planner is to assign rights and monitoring that the pricing rule is achieved by any agent.

The idea of introducing a parameter that modifies the budgetary set of consumers without participating in their preferences has been previously used in microeconomic theory to study other problems. For instance, in monetary economics a slack parameter is defined, usually called fiat money, whose sole role is to facilitate the exchange in the presence of "frictions" in the economy that make it difficult for agents to execute net-exchanges worth exactly zero. See [12] for more details on this relevant aspect of fiat money in the economy. See also [4] as a complementary reference.

Another example is one where the non-satiation assumption does not hold in the economy. If for any given price, some consumers wish to consume a commodity bundle in the interior of their budget set, the Walras equilibrium may fail to exist. In this case one may establish existence of an equilibrium by allowing for the possibility that some agents spend more than the value of their initial endowment. This generalization of the Walras equilibrium is called dividend equilibrium or equilibrium with slack (see [3], [5], [11], [18] and [16] among others).

The r-economy also has some similarities with the system of pollution rights to control emissions, where pollution rights are also tradable, however, here too, there are some fundamental differences. In the case of pollution rights, the price to trade rights is exogenously given by technical relationships, it affects only one good (more precisely, one bad) and directly affects the production of goods. Conversely, the r-economy endogenously defines consumption right prices, affecting all goods and is defined as an exchange economy without production being affected. See [19], [20] and [22] for more details on this type of model.

Moreover, our notion of consumption rights should not be confused with the concept used by Hammond in [13], defined as a right to "choose net trade vectors". It is also different to Aumann and Kurz's notion in [2], which represents the right to choose lump-sum transfers according to the individuals's or group's political power in the society. On the other hand, these concepts bear in common the fact that they are all defined from equity considerations, by means of political institutions.

The main difference of our approach compared with the above mentioned models is that in our model, in addition to the rights price, there are two different prices for each good in the market, with all of them being endogenously determined at the equilibrium. This introduces a second budget constraint, which combined with the usual wealth constraint, defines the budgetary set for any individual. Moreover, the possibility of trading rights links the two constraint, in a way that the individual faces a "flexible budgetary set". As a consequence, the demand for rights in the market is implicitly defined, as is the demand for goods.

In our mind, this approach adds a complementary market mechanism that allows an equilibrium allocation intended to comply with some exogenous equity criteria. In this way, a double objective -efficiency and equity- can be achieved by combining market mechanisms, each one specifically designed for each criteria but interacting without reducing their capability of reaching their underpinning objectives.

This paper is organized as follows. In Section 2 we introduce a simple and motivating example of the model, which is formalized in Section 3. In Section 4 we study the demand for consumption rights. The existence of equilibrium in the r-economy is proved in Section 5, whereas in Section 6 se study a slightly more general case where only a subset of markets are subject to rights. In Section 7 we analyze the redistribution aspects of the economy and finally, Section 8 is devoted to some final remarks on this work.

2 The model

2.1 Mathematical notation and preliminaries

For a matrix $A \in I\!R^{m\times n}$, its transpose is denoted as A^t and, when exists, its inverse by A^{-1}. A matrix $A \in I\!R^{m\times n}$ is said to be *positive* (resp. strictly positive), denoted $A \geq 0$ (resp. $A > 0$), if all of its elements are non negative (resp. strictly positive). The *spectral radius* of $A \in I\!R^{n\times n}$ is denoted by $\rho(A)$. The subset of M-matrices of dimension $n \times n$ is denoted by $M[n]$. Thus,

$$M[n] = \Big\{\mu I_n - A,\ A \geq 0, A \in I\!R^{n\times n},\ \mu \geq \rho(A)\Big\},$$

where $I_n \in I\!R^{n\times n}$ is the identity matrix. Finally $M^*[n] \subseteq M[n]$ denotes the subset of non-singular elements of $M[n]$. We refers to [6] for more details and properties of M-matrices. Finally, the inner product between $x,\ y \in I\!R^n$ is denoted as $x \cdot y$.

2.2 Consumption rights

Following the standard Arrow-Debreu model (see [1]), we assume that in the economy there are $\ell \in I\!N$ consumption goods and $m > 2$ consumers, indexed by $i \in I = \{1, 2, \cdots, m\}$, whose utility functions and initial endowments are given by $u_i : I\!R^\ell_+ \rightarrow I\!R$ and $\omega_i \in I\!R^\ell_+$ respectively. Define

$$\omega = \sum_{i\in I} \omega_i \in I\!R^\ell_{++}.$$

as the total initial resources of the economy. Thus an exchange economy is defined by

$$\mathcal{E} = ((u_i), (\omega_i))_{i\in I.}$$

Hereafter, we assume that utility functions are of class C^1, strictly quasi-concave and strictly increasing by components.

Any distribution of goods $\{x_i\}_{i\in I}$ is said to be a feasible allocation if

$$\sum_{i\in I} x_i = \sum_{i\in I} \omega_i.$$

In our model we will assume that, besides an initial endowment of resources, each consumer is initially assigned with a strictly positive amount of what we call *consumption right*, which generically is denoted by $r_i \in I\!R_+$. The exchange economy with consumption rights, called r-economy, is therefore defined by

$$\mathcal{E}_r = ((u_i), (\omega_i), (r_i))_{i\in I}\,.$$

We assume that consumption rights can be freely traded on the market, but they do not participate in the utility function; we denote by $q \in \mathbb{R}_+$ the price of consumption rights in the market.

Consumption rights modify the consumer's budget constraint in the following way. Assume that goods price is given by $p \in \mathbb{R}^\ell_+$ and that in the economy there are *transformation rates from goods to consumption rights*, which generically will be represented by a vector $s \in \mathbb{R}^\ell_{++}$. The solely role of vector s in the economy is to restrain the set of consumption possibilities of each agent, in such a way that with the initial consumption rights $r_i \in \mathbb{R}_+$ available for the individual $i \in I$, he can only consume those bundles $x_i \in \mathbb{R}^\ell_+$ that comply with

$$s \cdot x_i \leq r_i. \tag{1}$$

Inequality 1 represents a restriction for consumption expressed in terms of consumptions rights, which equivalently can be presented in terms of wealth using price q as follows

$$q\,(s \cdot x_i) \leq q r_i. \tag{2}$$

To finally determine the feasible consumption bundles for individual $i \in I$, condition (2) should be also considered along with the usual wealth constraint, that is,

$$p \cdot x_i \leq p \cdot \omega_i.$$

Now on, and this is a relevant point in our model, if the individual $i \in I$ may decide to trade $\delta \in \mathbb{R}$ consumption rights in the market, obtaining $r_i + \delta$ consumption rights, but his wealth is modified in $-q\delta$. Given this transaction of consumption rights, which ex post will be determined endogenously as a part of the equilibrium, the budgetary set of an individual $i \in I$ corresponds to

$$B(p, s, q, \omega_i, r_i, \delta) = \Big\{x \in \mathbb{R}^\ell_+ \mid p \cdot x \leq p \cdot \omega_i - q\delta,\ (qs) \cdot x \leq q r_i + q\delta\Big\}. \tag{3}$$

Thus, for a given $\delta \in \mathbb{R}$ and $i \in I$, consider now the following optimization problem

$$\mathcal{P}_i(\delta) \quad : \quad \begin{cases} \max & u_i(x) \\ \text{s.t} & x \in B(p, s, q, \omega_i, r_i, \delta). \end{cases}$$

Provided that $p,\ s \in \mathbb{R}^\ell_{++}$ and $q > 0$, we remark that argmax $\mathcal{P}_i(\delta)$ is non-empty if

$$\delta \in \Gamma_i = \left[-r_i, \frac{p \cdot \omega_i}{q}\right]. \tag{4}$$

Moreover, if condition (4) is satisfied, the solution of $\mathcal{P}_i(\delta)$ is unique. Let us denote it by $x_i(p, s, q, r_i, \delta)$ and define the *demand for consumption rights* as

$$\text{argmax}_{\delta \in \Gamma_i}\ \{u_i(x_i(p, s, q, r_i, \delta))\}.$$

Proposition 2.1 *For any consumer $i \in I$, if $p, s \in I\!R^{\ell}_{++}$, $q > 0$ and $\omega_i \in I\!R^{\ell}_{+}$, then the demand for consumption rights exists and it is unique.*

Proof. Given $i \in I$, the fact that demand for consumption rights is well defined is direct from both the continuity of the utility function and the compactness and non-emptiness of the budget set. Then, let $\delta_i \in \Gamma_i$ be a demand for rights at the indicated prices and let $x_i = x_i(p, s, q, r_i, \delta_i)$ be the corresponding demand for goods. Since the utility function is strictly increasing by component, it follows that at least one constraint of the consumer problem is binding at the optimum. Without loss of generality, let us assume that

$$p \cdot x_i = p \cdot \omega_i - q\delta_i, \quad s \cdot x_i < r_i + \delta_i.$$

Given $e_1 = (1, 0, 0, ..., 0) \in I\!R^{\ell}$, define

$$\epsilon = \frac{r_i + \delta_i - s \cdot x_i}{s \cdot e_1 + \frac{p \cdot e_1}{q}} > 0,$$

and given this, let $\bar{x}_i = x_i + \epsilon e_1$ and $\bar{\delta}_i = \delta_i - \frac{p \cdot e_1}{q}\epsilon$. It is direct that $\bar{x}_i$ and $\bar{\delta}_i$ comply with

$$p \cdot \bar{x}_i = p \cdot \omega_i - q\bar{\delta}_i, \quad s \cdot \bar{x}_i = r_i + \bar{\delta}_i,$$

and hence, from monotonicity of utility function, $u_i(\bar{x}_i) > u_i(x_i)$. Thus, δ_i would not be the consumer's demand for rights and therefore at the optimum both constraints are binding[2]. Consequently, from the fact that $p \cdot x_i = p \cdot \omega_i - q\delta_i$ and $s \cdot x_i = r_i + \delta_i$, we conclude that at the optimum satisfies that

$$(p + qs) \cdot x_i = p \cdot \omega_i + qr_i,$$

which finally allow us to deduce that the r-consumer's problem can be equivalently rewritten as

$$\begin{cases} \max \ u_i(x) \\ \text{s.t} \quad (p + qs) \cdot x = p \cdot \omega_i + qr_i. \end{cases}$$

Since the utility function is strictly quasi-concave, the solution to the above problem is unique, and so is the demand for rights. □

3 Equilibrium with a pricing rule

The equilibrium notion in the r-economy is simply a natural extension of the Walrasian equilibrium concept for the standard economy without consumption rights. Thus, we are concerning on prices p, s and q such that the respective demands (consumption rights and goods) at these prices are feasible allocations according to the notions given immediately.

[2]A similar conclusion can be obtained if we consider that $p \cdot x_i < p \cdot \omega_i - q\delta_i$ and $s \cdot x_i = r_i + \delta_i$.

Definition 3.1 A consumption rights distribution $\{\delta_i\}_{i\in I}$ is said to be feasible if

$$\sum_{i\in I} \delta_i = 0.$$

Definition 3.2 Prices $(p^*, s^*, q^*) \in I\!R^{2\ell}_+ \times I\!R_+$ are said to be an equilibrium price for the economy $\mathcal{E}_r$ if there exists a feasible distribution of goods $\{x_i^*\}_{i\in I}$ and a feasible distribution of right exchanges $\{\delta_i^*\}_{i\in I}$, such that

(a) for each $i \in I$, δ_i^* is the rights demand at prices (p^*, s^*, q^*),

(b) for each $i \in I$, $x_i^* = x_i(p^*, s^*, q^*, r_i, \delta_i^*)$.

The tupla $(p^*, s^*, q^*, (x_i^*)_{i\in I}, (\delta_i^*)_{i\in I})$ constitutes what we call a competitive equilibrium for the economy $\mathcal{E}_r$.

Observe that the number of unknowns involved in the r-equilibrium definition is $2\ell + 1$ (components of prices p, s and q) and, from the feasibility condition in goods and transaction of consumption rights, the number of equations that define it is $\ell + 1$. Hence, there is a fundamental indeterminacy in our equilibrium concept. In order to overcome this problem, we are enforced to define what we call a pricing rule. Precisely, the pricing rule and the assignment of consumption rights to consumers will define the policy maker instruments needed to reach any social objective in the economy.

The simplest way to define a pricing rule is by means of a linear relation among prices as we define in the following.

Definition 3.3 A linear pricing rule in the r-economy will be a relation

$$s = \frac{1}{q} Ap$$

with $A \in I\!R^{\ell\times\ell}$ a given matrix[3].

Given a linear pricing rule as before we can reformulate the r-equilibrium notion above introduced. Considering that if $(p^*, s^*, q^*, (x_i^*)_{i\in I}, (\delta_i^*)_{i\in i})$ is an equilibrium of $\mathcal{E}_r$, then for each $i \in I$, $s^* \cdot x_i^* = r_i + \delta_i^*$ and then, $s^* \cdot \sum\limits_{i\in I} x_i^* = \sum\limits_{i\in I} r_i + \sum\limits_{i\in I} \delta_i^* = \sum\limits_{i\in I} r_i$. Thus, if we define $R = \sum_{i\in I} r_i$, condition $s^* \cdot \omega = R$ should always be verified at any equilibrium price. On the other hand, note that under the linear pricing rule as above, the consumer's budgetary restriction of individual $i \in I$ can be rewritten as

$$(p + Ap) \cdot x = p \cdot \omega_i + qr_i \quad \Leftrightarrow \quad [(I + A)p] \cdot x = p \cdot \omega_i + qr_i, \tag{5}$$

and then, given previous considerations, we can re-write the consumer's problem in the r-economy as

[3] Parameter q is introduced here just for simplicity.

$$\begin{cases} \max & u_i(x) \\ \text{s.t} & [(I+A)p] \cdot x = p \cdot \omega_i + qr_i. \end{cases}$$

If we denote by $x_i(A,p,q)$ the respective demand for the previous problem, then we can readily deduce that $(p^*,q^*) \in \mathbb{R}^\ell \times \mathbb{R}$ is an equilibrium price for the r-economy if and only if

(a) $\sum\limits_{i\in I} x_i(A,p^*,q^*) = \omega$,

(b) $s^* \cdot \omega = R$ (or equivalently $Ap^* = Rq^*$).

Indeed, if (p^*,q^*) is an equilibrium price as before, given

$$s^* = \frac{1}{q^*}Ap^*, \;\; \delta_i^* = s^* \cdot x_i(A,p^*,q^*) - r_i, \;\; x_i^* = x_i(A,p^*,q^*),$$

it is easy to check that $(p^*,s^*,q^*,(x_i^*)_{i\in I},(\delta_i^*)_{i\in i})$ is an r-equilibrium of $\mathcal{E}_r$.

If we assume that $(I+A)$ is non-singular and defining $\pi = (I+A)p$, condition (5) can be re-written as

$$\pi \cdot x = \pi \cdot \left[(I+A)^{-t}\omega_i\right] + qr_i,$$

and then, due to $s \cdot \omega = R$ it follows that

$$q = \frac{\pi \cdot [(I+A)^{-t}A^t\omega]}{R}. \tag{6}$$

Thus, if we define $\rho_i = \frac{r_i}{R}$ and $\rho = (\rho_i) \in \Delta_m$, the consumer's problem for individual $i \in I$ can be finally expressed as

$$\begin{cases} \max & u_i(x) \\ \text{s.t} & \pi \cdot x = \pi \cdot \omega_i(\rho,A) \end{cases} \tag{7}$$

with

$$\omega_i(\rho,A) = (I+A)^{-t}\left(\omega_i + \rho_i A^t\omega\right). \tag{8}$$

Note that $\sum\limits_{i\in I} \omega_i(\rho,A) = \omega$. Thus, in order to guarantee that $\{\omega_i(\rho,A)\}_{i\in I}$ constitute a redistribution of initial endowments in the economy we must verify that for each $i \in I$

$$\omega_i(\rho,A) \in \mathbb{R}^\ell_+. \tag{9}$$

If condition (9) were true, from standard results in general equilibrium theory we know that there exists an equilibrium price $\pi(\rho,A) \in \mathbb{R}^\ell_{++}$ for the economy. Denoting $x_i(\pi(\rho,A),\omega_i(\rho,A))$ the respective equilibrium allocations, it is easy to check that $(p^*,s^*,q^*,(x_i^*)_{i\in I},(\delta_i^*)_{i\in I})$ defined by

$$p^* = (I+A)^{-1}\pi(\rho,A) \qquad q^* = \frac{\pi(\rho,A) \cdot [(I+A)^{-t}A^t\omega]}{R}$$

$$s^* = \frac{1}{q^*}Ap^* \qquad x_i^* = x_i^*(\pi(\rho, A), \omega_i(\rho, A)),$$

and $\delta_i^* = s^* \cdot x_i^* - r_i$ constitutes an r-equilibrium for $\mathcal{E}_r$, provides that $q^* > 0$.

The fact that $q^* > 0$ and $\omega_i(\rho, A) \in I\!R_+^\ell$ not only depends on the non-singularity of $I + A$ but also on the positiveness of matrix $(I + A)^{-1}$.

Proposition 3.1 *For each $i \in I$, $\omega_i(\rho, A) \in I\!R_{++}^\ell$ and $q^* > 0$ if any of the following condition is satisfied*

(a) $A \in M^*[\ell]$, with $A > 0$.

(b) $A = Diag[\sigma_i]$: diagonal matrix of $\sigma_i > 0$

(c) Given $1 \leq k \leq \ell$, $\omega_i(-k) = (\omega_i^{k+1}, \cdots, \omega_i^\ell) \in I\!R_{++}^{\ell-k}$ and

$$A = \begin{bmatrix} A_k & 0 \\ 0 & 0 \end{bmatrix}$$

with $A_k \in M^*[k]$ such that $A_k\omega(k) \in I\!R_{++}^\ell$, with $\omega(k) = (\omega_1, \omega_2, \cdots, \omega_k) \in I\!R^k$.

Proof. We recall that $A \in M^*[\ell]$ if A is non-singular and there exist $B \geq 0$ and $\mu > \rho(B)$ such that

$$A = \mu I - B.$$

Since $\rho(B) < \mu$ follows that $I + A$ is non-singular (and so $(I + A)^t$) and therefore, considering that

$$(I + A)^{-t} = (I + A^t)^{-1} = ((1 + \mu)I - B^t)^{-1} = \frac{1}{1 + \mu} \lim_{n \to +\infty} \sum_{k=0}^{n} \left(\frac{B^t}{1 + \mu}\right)^k = \frac{1}{1 + \mu}I + \Gamma$$

with $\Gamma \geq 0$, we conclude that under condition (a)

$$\omega_i(\rho, A) = \left[\frac{1}{1 + \mu}I + \Gamma\right](\omega_i + \rho_i A\omega) \in I\!R_{++}^\ell.$$

Now on, if we define $\mu = \sum_{i \in I} \sigma_i$ and $B = Diag[\mu - \sigma_i]$ then $A = \mu I - B$, we have that $A = Diag[\sigma_i]$ satisfies the hypotheses of case (b) and then the result is verified.

For case (c), note that

$$\omega_i(\rho, A) = (I + A)^{-t}\left(\omega_i + \rho_i A^t\omega\right) = (\omega_i(\rho, A)(k), \omega_i(\rho, A)(-k))$$

with

$$\omega_i(\rho, A)(k) = (I + A_k)^{-t}\left(\omega_i(k) + \rho_i A_k^t\omega(k)\right) \in I\!R^k, \quad \omega_i(\rho, A)(-k) = \omega_i(-k) \in I\!R^{\ell-k}.$$

Thus, from condition (a) and the hypotheses (c) we have that $\omega_i(\rho, A)(k) \in I\!R^k_{++}$ and from the hypotheses (c) again we conclude that $\omega_i(\rho, A)(-k) \in I\!R^{\ell-k}_{++}$, which ends the proof. $\square$

Finally, since $\pi^* \in I\!R^{\ell}_{++}$, $A \geq 0$ and $(I + A)^{-1} \geq 0$ as yet obtained, then is direct that $q^* > 0$. $\square$

Remark 3.1 The simplest pricing rule one can consider is $A = \sigma I$, for some $\sigma > 0$. In such case, given $\rho \in \Delta_m$ and

$$\lambda = \frac{1}{1+\sigma} \in [0, 1],$$

we can readily deduce that

$$\omega_i(\rho, A) = \lambda\omega_i + (1 - \lambda)\rho_i\omega_i.$$

This case can be interpreted either as a (i) lump-sum among individuals or (ii) as the outcome of a tax inflation mechanism. For the first case, the tax device consist on collecting $(1 - \lambda)$ of the initial resources of the economy and give then back to the individuals according to the percentages of consumption rights initially assigned to them. As a tax inflation outcome, consider the equilibrium price without intervention p_c and the equilibrium price with intervention $\widetilde{p}$, then define

$$M_i = [\lambda\widetilde{p} - p_c] \cdot \omega_i + (1 - \lambda)\rho_i\widetilde{q}\sigma R > 0$$

as money in the initial non-intervened situation. Then the consumer's problem is

$$\begin{cases} \max \ \ u_i(x) \\ \text{s.t} \quad p \cdot x = p \cdot \omega_i + M_i. \end{cases} \tag{10}$$

Then the equilibrium price is $\widetilde{p}$ and the demands are those obtained in the intervened economy as described.

4 The policy maker problem regarding wealth distribution: a simple example

Consider an economy with m individual indexed by $i \in I = \{1, 2, \cdots, m\}$ and two goods (i.e $\ell = 2$). Assume that their utility functions are given by $u_i(x, y) = x^{\alpha_i}y^{1-\alpha_i}$ and the respective initial endowments are $\omega_i = (\omega_{i1}, \omega_{i2})$. Define

$$\omega = (\omega_1, \omega_2) = \sum_{i \in I} \omega_i \in I\!R^2.$$

In this case, considering good one as the numerary, the competitive equilibrium price is $\mathbf{p_c} = (1, p_c)$ with

$$p_c = \frac{\sum\limits_{i \in I}(1 - \alpha_i)\omega_{i1}}{\sum\limits_{i \in I} \alpha_i\omega_{i2}}.$$

Assume now that $A = \sigma I$ and $\lambda = \frac{\sigma}{1+\sigma} \in]0,1[$. Given $\rho \in \Delta_m$ and

$$\omega_i(\rho, A) = (\widetilde{\omega}_{i1}, \widetilde{\omega}_{i2}) = \lambda \omega_i + (1-\lambda)\rho_i \omega$$

the equilibrium price in this r-economy is given by $\widetilde{\mathbf{p}} = (1, \widetilde{p})$ with

$$\widetilde{p} = \frac{\sum\limits_{i \in I} (1-\alpha_i)\widetilde{\omega}_{i1}}{\sum\limits_{i \in I} \alpha_i \widetilde{\omega}_{i2}}.$$

In the original economy, the wealth of an individual $i \in I$ is given by

$$W_i^c = \mathbf{p}_c \cdot \omega_i = \omega_{i1} + p_c\, \omega_{i2},$$

whereas in the r-economy this value is

$$W_i^r = \widetilde{\mathbf{p}} \cdot \omega_i(\rho, A) = \widetilde{\omega}_{i1} + \widetilde{p}\, \widetilde{\omega}_{i2}$$

If we define the total wealth of the r-economy (economy) as $TW^r = \widetilde{\mathbf{p}} \cdot \omega$ ($TW^c = \mathbf{p}^c \cdot \omega$), the *relative wealth of an individual* $i \in I$ in the r-economy is given by

$$RW_i^r = \frac{W_i^r}{TW^r} = \lambda \left[\frac{\widetilde{\mathbf{p}} \cdot \omega_i}{TW^r} \right] + (1-\lambda)\rho_i. \tag{11}$$

Now on, when all individuals have the same utility function, let say $\alpha_i = \alpha_j$ $(= \alpha)$, $\forall i, j$, we can check that[4]

$$\widetilde{\mathbf{p}} = \mathbf{p_c}, \tag{12}$$

which implies that $TW^r = TW^c$ and $\widetilde{\mathbf{p}} \cdot \omega_i = W_i^c$. Thus,

$$RW_i^r = \lambda \left[\frac{W_i^c}{TW^c} \right] + (1-\lambda)\rho_i \equiv \lambda RW_i^c + (1-\lambda)\ \rho_i. \tag{13}$$

Thus, from all foregoing, a desirable wealth distribution $w = (w_i) \in \Delta_m$ it can be attained in the following way: define $\lambda^* \in]0,1[$ such that

$$\lambda^* \max_{i \in I} \{RW_i^c\} < \min_{i \in I} \{w_i\}.$$

This value of λ^* corresponds to assume $A = \sigma^* I$ with

$$\sigma^* = \frac{\lambda^*}{1-\lambda^*}$$

the linear pricing rule. Thus, if we define

$$\rho_i^* = \frac{w_i - \lambda^* \left(RW_i^c\right)}{1-\lambda^*} \in]0,1[$$

and assign consumption rights according to these percentages, then the desired wealth distribution is attained.

[4]Condition (12) holds true if, for instance, there are ℓ goods in the economy and the identical preferences of all consumers is given by a Cobb-Douglas homogenous of degree one or by a CES utility function.

5 Decentralization

A central question dealing with public choice arises when we try to characterize which Pareto allocations can be decentralized through the competitive process. We discuss this very relevant issue for the r-economy whenever we are interested in implementing a social policy defined ex ante.

In the following, just to illustrate the method, we will consider the simplest pricing rule

$$p = qs. \tag{14}$$

The redistribution mechanism inherent to the pricing rule as before can not reach any point on the contract curve, because, as we already know, the lump-sum transfer consists of collects only half of the total resources in the economy and assigns them to individuals according to the percentage of rights they own. Thus, in the extreme case an individual can be "taxed" with only 50% of his resources, which is not enough to reach any Pareto allocation.

The following figure of an Edgeworth box, with individual's initial endowments given by ω_1 and ω_2, illustrates to us that there are some points on the contract curve (passing trough $CEDF$) that can not be decentralized with any assignment of positive consumption rights according to the rule previously detailed. Point A corresponds to $\omega_1/2$, that is, when $r_1 = 0$ (and therefore $r_2 = R$), whereas point D is $\omega_1/2 + [\omega_1/2 + \omega_2/2]$, that is, when $r_1 = R$. Just to illustrate the idea, suppose that all supporting lines of any Pareto belonging in the portion CED of the contract curve are included in the shadowed region in Figure 2 (which is delimited by the supporting lines passing trough A and D). In such case, only Pareto points belonging on this portion of the curve can be decentralized by an adequate amount of rights. This is the case, for instance, with allocation E. Conversely, the Pareto allocation F can not be decentralized because the intersection between its supporting line defined by price p_F does not intersect the shadowed region.

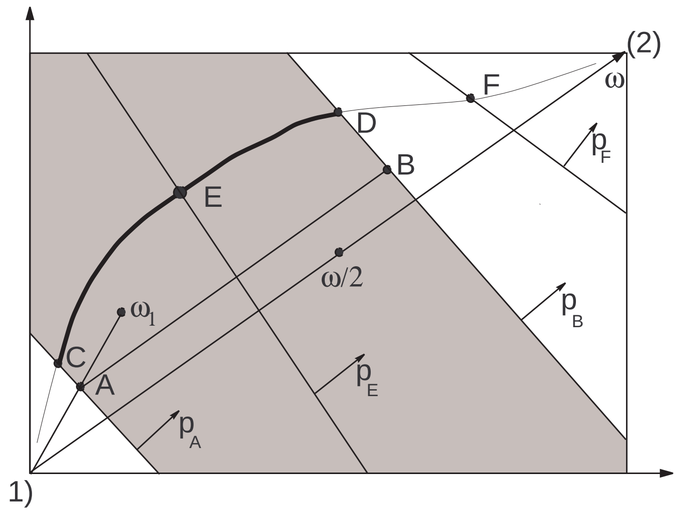

Figure 2.

Thus, the question of which are the Pareto allocations that can be decentralized using consumption rights to implement the lump-sum transfers, can be now answered. From the previous figure, and restricted to the case of positive rights and a pricing rule as (14), this procedure can not decentralize Pareto optimum allocations that correspond to a "radical redistribution" of wealth, that is, all that make a "poor individual" to become "rich" and a rich to become a poor. This is the case, for instance, with allocation F in the above figure.

The procedure yet described it reduces the wealth inequality but prohibits extreme progressive outcomes. This result is in the line with Roemer's notions of equalitarian societies (see [23]), since they also correspond to allocations located in portion CED of the contract curve.

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