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Pollution in times of economic uncertainty: A perverse tragedy of the commons?

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ABSTRACT

We explore the effects of economic uncertainty on environmental pollution. We show conditions under which an increase of economic uncertainty raises pollution. The results depend on the consumers' level of risk aversion and prudence behavior, as well as on the elasticity of substitution between pollution and conventional inputs. Under several widely used specifications for preferences and production technologies, we show that, given available empirical evidence about the parameters characterizing these specifications, a damning vicious cycle between increasing economic uncertainty and pollution is likely to occur.

Keywords: Higher-order risk attitudes; risk aversion; prudence; cross prudence; elasticity of factor substitution.

JEL Classification: C62; D81; O13; Q52

1. Introduction

In recent years, many countries are facing two important new developments. First, the effect of climate change -mostly derived from ever expanding global environmental pollution- which has apparently caused greater economic uncertainty. Second, the massive sanitary crisis caused by the Covid-19 virus. While the short run pollution-reducing impact of the sanitary crisis is obvious (associated mostly with a drastic cut in economic activity), its medium- and long-term effects through a possible dramatic increase in uncertainty are not as clear.

This paper deals with the environmental consequences of increased uncertainty. The importance of this analysis is underlined by the potential vicious cycle that may be caused if the environment is negatively affected by increasing uncertainty. Greater pollution, especially through its climate change impact, is likely to cause more economic uncertainty and therefore if greater uncertainty causes further pollution and hence climate worsening, we may have a damning vicious circle. Consequently, the difficulties of dealing with climate change may be even worse than what is usually envisioned.

Here we use the definition from Dardanoni (1988) to differentiate risk from uncertainty. Risk is defined as a decrease in the mean of a random variable of interest (e.g., a decrease in the expected growth rate of the economy), the so called First-Order Risk. On the other hand, uncertainty is defined as a mean-preserving spread in the probability distribution of the economic status often called Second-Order Risk.

Most studies of pollution have focused on the environmental effects of economic growth (e.g., the so-called Kuznets curve)[1]. However, in view of the stylized facts enumerated at the outset of this paper, it appears that the key determinant of pollution is likely to be associated not only with changes in long term economic growth but especially with greater risk and uncertainty.

The relationship between economy-wide uncertainty and environmental pollution has been the object of only few studies. From an empirical point of view, the evidence at the micro level suggests that environmental degradation is sometimes used as a hedge against income risk. Dasgupta and Maler (1993, p.19) argue that local commons "…provide the rural poor with partial protection in times of unusual economic stress. For landless people, they may be the only non-human asset at their disposal." In addition, Bromley and Chavas (1989, p.130) suggest that non-exclusive property rights can "be seen as an integral part of risk-sharing strategies." Both papers argue that the absence of property rights allows poor people to better deal with income variability, albeit possibly increasing environmental degradation.

In the same line of research, Baland and Francois (2005) analyze the negative welfare impact of both the insurance value of privatization of common goods and the often-encountered resistance to efficiency reforms regarding village level landholding and forest. Using data from a survey conducted in the Brazilian Amazon, Pattanayak and Sills (2001) suggest that households rely on forests to mitigate agricultural income risk. Delacote (2007, 2009) addresses deforestation due to the use of forest products as safety nets by poor agricultural households facing increasing income uncertainty.

Since the pathbreaking paper by Kimball (1990) on precautionary saving, all the papers investigating the effect of uncertainty on various endogenous variables find that there is a

[1] For a review of the Environmental Kuznets Curve literature see Dinda (2004) and Usenata (2018).

threshold value for the *coefficient of relative prudence* that determines much of the qualitative effects of increasing uncertainty (Kimball, 1990; Eeckhoudt and Schlesinger, 2008; Chiu and Eeckhoudt, 2010).[2]

At the economy-wide level, we do not have, however, an equivalent theory of the effect of economic uncertainty on pollution,[3] thus we do not know if the same threshold value for relative prudence applies.

More importantly, from a policy viewpoint the evidence of the impact of economy-wide uncertainty on environmental pollution is rather scarce, and the existing studies present conflicting results. Feng et al (2015) argue that reductions in carbon dioxide emissions in the US between 2007 and 2012 were more likely caused by economic slowdown and crisis rather than green technological innovation. In contrast, O´Riordan and Turner (1984) argue that economic uncertainty creates a danger of increasing industrial pollution owing to greater incentives by firms to evade environmental regulations or even by a relaxation of regulations by governments. In the same vein, Khan (2019) presents evidence for Pakistan showing that macroeconomic uncertainty increases aggregate pollution emissions. He concludes that "a stable macroeconomic environment is also necessary for economies in order to reduce pollution emissions and make the transition from polluted to environment-friendly energy production technologies".

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[2] The coefficient of relative prudence is the elasticity of the second derivative of the marginal utility with respect to income.

[3] This is rather surprising given the fact that income and risk are central issues to development (Elberts et al, 2007). A reason may be that general equilibrium models of economic growth and the environment use deterministic models of exogenous economic growth amended to include the environment (Elbers et al, 2007). These models use the framework for the study of the Environmental Kuznets Curve (EKC) (Lopez, 1994; Pastén and Figueroa, 2012). This analysis can be regarded as a steady state deterministic one. Nevertheless, after the original work of Levhari and Srinivasan (1969), and the work of Brock and Mirman (1972), interest in economic growth and risk seems to have revived (Binder and Pesaran, 1999; De Hek, 1999).

Our analysis is a first attempt to theoretically study the direct relationship between risk and uncertainty on the one hand and environmental pollution on the other using an economy-wide framework. We show the conditions under which an increase of economic uncertainty could worsen the environment. We show that an increase in economic uncertainty is more likely to increase pollution when consumers risk aversion, prudence coefficient, and the elasticity of substitution in production between natural and man-made inputs are high. That is, the above-mentioned damning vicious circle between raising risk and uncertainty and pollution is certainly possible.[4]

We assume that pollution is optimally determined at the country level through either a planner or a competitive market that internalizes the social cost of pollution. However, optimality at the country level does not imply optimal emissions at the world level. A key question that we examine below is the following: Will countries optimizing their emissions considering their own welfare increase emissions in the face of greater uncertainty caused by, for example, climate change? If the answer is yes, the vicious cycle just mentioned may occur. Pollution would bring about more economic risk and uncertainty (through climate change, for example) and increased risk and uncertainty would bring in more pollution, which in turn (e.g., by further deepening climate change) would worsen uncertainty and risk, and so on.

We build our analysis on a rich literature studying risk and uncertainty (Howarth, 2003; Aalbers, 2009; Gollier, 2010; Weitzman (2011, 2013), Gollier, 2019). Our contribution is closely related to Gollier (2010) who studies how natural capital should be discounted over

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[4] However, it is possible that a particularly profound increase of economic uncertainty (as the one that the current sanitary crisis could cause) may be accompanied by such a large reduction of economic growth that the effect of the latter phenomenon more than off-set the direct effect of uncertainty, thus causing pollution to actually decline even if the above-mentioned conditions do occur. However, this may be valid only for the short run when the full impact of the crisis is felt. Once the initial impact of the shock dissipates the most important remaining sequel of the crisis is likely to be the increased uncertainty.

time, showing that the rate at which natural capital is discounted (or the economic value of physical capital) increases with uncertainty in the elasticity of substitution.[5]

2. **The model**

We assume that $u(c, q)$ is a country welfare function, where c is consumption of goods and q is the quality of the environment, and the function $u(c, q)$ is assumed strictly increasing, concave and thrice continuously differentiable. The aggregate budget constraint is given by $c = f(k, x)$, with $f(k, x)$ being the aggregate production function; k is man-made capital, broadly defined to include physical, human, and technology capital, and x is the flow of pollution.[6]

The price of pollution is assumed endogenous and optimally determined by a social planner or by a perfect market. For a unit of the total endowment of environmental quality, pollution x is given by $1 - q$. We restrict our analysis to linearly homogenous production functions. The production function is assumed increasing and concave in the factors of production and that the Inada conditions hold (also, the cross derivative is positive). Following Tahvonen and Kuuluvainen (1993) and Lopez (1994), pollution can be regarded as a factor of production.

Under certainty, the maximization of $u(\cdot)$ with respect to x yields the optimal level of aggregate pollution under the assumption that all (country) effects of pollution are internalized (Lopez, 1994).

$Max\ u(f(k, x), 1 - x)$ [Problem 1]

[5] We depart from Gollier (2010) and Gollier (2019) by explicitly including pollution in the utility function of the representative consumer.

[6] This model is a simplified general equilibrium model in the spirit of Lopez (1994). Following the author, k_i is an aggregator function of capital (k_i'), labor (l_i) and technology (t) for each industry i in the form $k_i = k_i(k_i', l_i;t)$. With standard assumptions, aggregation theorems allow us to define $f(k, x)$ in terms of aggregate conventional factors, and aggregate emissions x rather than firm specific emission x_i. Thus, $f(k, x)$ can be interpreted as the equivalent to national income.

The necessary and sufficient conditions for a maximum are[7]:

$$H(k,x) \equiv f_x(k,x_0)u_c(f(k,x_0),1-x_0) - u_q(f(k,x_0),1-x_0) = 0 \quad (1)$$

$$H_x(k,x) = u_{qq} - 2f_x u_{qc} + f_x^2 u_{cc} + u_c f_{xx} < 0 \quad (2)$$

The optimal level of pollution x_0 is determined from Eq. 1.

Turning now to the case of risk and uncertainty, we assume that k is the mean of the random variable $\tilde{k} \in [\underline{k}, \overline{k}]$. Therefore, income is given by $\tilde{y} = f(\tilde{k}, x)$, implying that the stochastic nature of income derives from the stochastic nature of $\tilde{k}$. That is, we postulate that the effect of risk and uncertainty takes place through the variable $\tilde{k}$ which considers all forms of capital stocks. Climate change, for example, may cause greater instability by inducing more frequent droughts, hurricanes and other climatic phenomena that may threaten the capital stocks of the economy, thus reducing their expected level and increasing their spread. Increased risk and uncertainty in $\tilde{k}$ is then transmitted into a higher risk and uncertainty for income ($\tilde{y}$) as well.

Now, the representative agent will choose x to maximize

$$Max\ Eu\big(f(\tilde{k},x),1-x\big). \quad \text{[Problem 2]}$$

The necessary and sufficient conditions for a maximum in Problem 2, assuming an interior solution, are given by

$$E(H) = Eu_c\big(f(\tilde{k},x^*),1-x^*\big)f_x(\tilde{k},x^*) - Eu_q\big(f(\tilde{k},x^*),1-x^*\big) = 0 \quad (3)$$

[7] Subscripts denote partial derivatives with respect the respective arguments (one subscript denotes first derivatives and two or more subscripts denote second or higher order derivatives).

$$E(H_x) = Eu_{qq} - 2Ef_x u_{cq} + Ef_x{}^2 u_{cc} + Eu_c f_{xx} < 0 \quad (4)$$

We consider next two comparative statics exercises concerning changes in the distribution of $\tilde{k}$: the effect of (i) First-Order Risk increase, and of (ii) Second-Order Risk increase.

2.1 Pollution and Risk

The effect of first-order risk increase (a leftward shift of the distribution of $\tilde{k}$) on the optimal choice of x is qualitatively the same as the comparative static prediction derived from the certainty case (See Appendix A). Thus, for a first-order risk increase in $\tilde{k}$, pollution unambiguously will increase (decrease) if x is everywhere decreasing (increasing) in k under certainty (Dardanoni, 1988).

Totally differentiating Eq. (1) with respect to k and x shows that pollution increases (decreases) if,

$$\frac{\partial \mathrm{x}}{\partial \mathrm{k}} = -\frac{H_k}{H_x} = -\frac{f_{xk}u_c + f_x u_{cc} f_k - u_{qc} f_k}{H_x} < (>) 0 \quad (5)$$

Or, equivalently, given that $H_x < 0$, if

$$H_k = f_{xk}u_c + f_x u_{cc} f_k - u_{qc} f_k < (>) 0 \quad (6)$$

Define $\eta \equiv -\frac{u_{cc}}{u_c} c$ as the coefficient of relative risk aversion (RRA); $\gamma \equiv -\frac{u_{qc}}{u_q} c$, the coefficient of relative correlation aversion (Eeckhoudt, Rey and Schlesinger, 2007; Gollier, 2010); and $\sigma \equiv \frac{f_x f_k}{f f_{xk}}$, as the elasticity of factor substitution between pollution x and factor k.

Then, x decreases (increases) in the mean of k if and only if,

$$H_k(k, x) = \varphi(k, x)\left(\frac{1}{\sigma} - \eta + \gamma\right) < (>) 0 \quad (7)$$

where $\varphi(k,x) \equiv \frac{f_x f_k u_c}{f}$ >0

From Eq.7, the following proposition follows:

Proposition 1 *First-Degree Risk Increase (a reduction in the mean of* $\tilde{k}$) *leads to pollution increases (decreases) iff,*

$$\eta - \gamma > (<) \frac{1}{\sigma} \tag{8}$$

Proof. *See Appendix A.*

For an individual who is correlation averse, γ is positive, because in this case $u_{qc} < 0$. For an individual who is correlation loving, γ is negative and $u_{qc} > 0$. Someone who is correlation averse prefers less environmental quality (more pollution) in a state with a higher level of income because correlation-aversion implies that a higher level of income mitigates the pain of a reduction in environmental quality. On the other hand, for a correlation-loving individual, the ordering of the above preferences is reversed, because he/she enjoys consumption less if he/she lives in a dirty environment (Eeckhoudt, Rey and Schlesinger, 2007).

We use an adjusted coefficient of relative risk aversion given by $\eta^A \equiv \eta - \gamma$, such that, the RRA is adjusted downward if people is relative correlation averse, and is adjusted upward if people is relative correlation loving. For additive preferences, $\eta^A = \eta$. Thus, pollution increases (decreases) with First-Degree Risk increase, whenever,

$$\eta^A > (<) \frac{1}{\sigma} \tag{9}$$

The intuition for Eq. 9, is the following: Facing a First Degree-Risk increase, if consumers are relative risk averse, their willingness to pay for environmental goods and services decreases (the supply price of pollution falls). In fact, the larger is η^A the greater will be the reduction of the supply price of pollution. However, firms will also reduce their demand price for pollution because the marginal product of pollution falls due to the reduction in the mean of k. The decrease in their demand price depends on the elasticity of substitution between pollution and capital. If the elasticity of substitution is large the marginal product of pollution will fall little and vice versa. If, for example, the production function were linear (infinite elasticity of substitution) then the marginal product of pollution would not be affected at all and, hence, the only effect left will be the reduced supply price of pollution causing it to expand.

2.2 Uncertainty

The most interesting case, however, is the effect of uncertainty (second order risk increase) on $\tilde{k}$. In a mean-preserving spread (MPS) of the distribution of $\tilde{k}$, the mean of $\tilde{k}$ is not affected, only its spread is. However, a MPS in $\tilde{k}$, is not necessarily a MPS in $\tilde{f}$. In fact, if the production function is concave, a mean preserving spread in $\tilde{k}$ results in a second order stochastic deterioration in $\tilde{f}$. Thus, unlike the distribution of $\tilde{k}$ which preserves its mean, the mean of $\tilde{f}$ falls, because strict concavity of f in k means $Ef(\tilde{k}, x) < f(k, x)$.

Therefore, in studying increasing uncertainty via a MPS in $\tilde{k}$ we are also implicitly analyzing the effects of an increase in the dispersion of $\tilde{f}$ in conjunction with a fall in its mean. Thus, we analyze the effect of stagnation and economic uncertainty at the same time. To reduce algebraic clutter, we henceforth assume that $u_{qc} = 0$ (although in Appendix C we relax this assumption).

Proposition 2. *Pollution increases (decreases) with a Second Order Risk increase in $\tilde{k}$ if and only if,*

$$H_{kk}(k, x) = 2f_{xk}u_{cc}f_k + f_x u_{ccc}f_k^2 + f_{xkk}u_c + f_x u_{cc}f_{kk} > (<)\ 0 \qquad (10)$$

Proof. *See Appendix B.*

First, some definitions: $P \equiv -\mathrm{c}\mathrm{u}_{\mathrm{ccc}}/\mathrm{u}_{\mathrm{cc}} > 0$, the *coefficient of relative prudence* in preferences, $\delta \equiv -\mathrm{kf}_{\mathrm{kk}}/\mathrm{f}_{\mathrm{k}} > 0$, the *degree of curvature in the production function*, $\alpha \equiv \mathrm{kf}_{\mathrm{k}}/\mathrm{f}$, is the *share of the risky factor k in the production* function, and, $\theta \equiv -\mathrm{f}_{\mathrm{xkk}}\mathrm{f}/\mathrm{f}_{\mathrm{xk}}\,\mathrm{f}_{\mathrm{k}}$, is what we have called, the *coefficient of relative cross-prudence in production*. The latter coefficient indicates by how much the expected value of marginal product of pollution ($E\mathrm{f}_{\mathrm{x}}$) increases or decreases when uncertainty increases.[8] If $\mathrm{f}_{\mathrm{xkk}} < 0$ then the marginal product of pollution is decreasing in the spread of $\tilde{k}$. This is probably the most plausible case, which means that θ may be assumed to be positive.

In Appendix B we show that using Eq. 10 pollution increases (decreases) with a Second Order Risk increase in $\tilde{k}$ if and only if

$$H_{kk} = \omega(k,x)\left[\eta\left(P - \frac{2}{\sigma}\right) - \frac{\theta}{\sigma} + \eta\frac{\delta}{\alpha}\right] > (<)0 \qquad (11)$$

where $\omega(k,x) = \frac{u_c {f_k}^2 f_x}{f^2} > 0$

Eq.11 shows three separate effects on pollution caused by increased uncertainty on $\tilde{k}$. The first term inside the square brackets, $P - (2/\sigma)$, is positive if relative prudence and the elasticity of substitution in production are both high enough. We can further explore the sign of $\left(P - (2/\sigma)\right)$ through the following expression:

$$sign\ of\ \left(P - (2/\sigma)\right) = sign\ of\ \left(u_{ccc}c + (2u_{cc}/\sigma)\right) \qquad (12)$$

The first term in the right-hand side of Eq. 12 is positive if the consumer is prudent ($u_{ccc} > 0$), this term tends to increase pollution. When people are prudent, they want to better deal with

[8] This can be seen by taking a Taylor series expansion of f_x around the mean of $\tilde{k}$ and taking its expected value.

uncertainty by shifting the aggregate income distribution upwards, which is possible by increasing pollution. Or, equivalently, the expected marginal utility of consumption of prudent people increases when uncertainty increases, which causes a fall in the supply price of pollution thus inducing greater pollution (remember that the expected price of pollution is decreasing in the expected marginal utility of consumption); they try to mitigate the detrimental effect of uncertainty by increasing pollution in a similar way as prudent agents increase the saving rate to deal with uncertainty in future income (Kimball, 1990).

This incremental effect on pollution is partially offset by the second term on the right-hand side of Eq. 12 ($2u_{cc}/\sigma$), which is negative under risk aversion and tends to reduce pollution. Since capital and pollution are gross complement inputs, and since capital is risky, an increase in risk tends to reduce the expected marginal product of pollution, which make pollution undesirable for risk averse agents. However, this compensating effect is likely to be small, especially if the elasticity of factor substitution is high.

The second term in Eq. 11, $-\left(\frac{\theta}{\sigma}\right)$, can be either positive or negative depending on the sign of f_{xkk}.This term is a type of *cross prudence in production*. The expected value of f_x -i.e. the expected marginal product from pollution to firms- can shift downwards or upwards with greater economic uncertainty in $\tilde{k}$. If $f_{xkk} < 0$, then Ef_x shifts downward and thus this effect is pollution-decreasing. [9] The opposite happens if $f_{xkk} > 0$. The relative cross-prudence coefficient in production, θ, reflects the magnitude of this effect. A cross prudent firm ($\theta > 0$)

[9] If the price w that polluters pay for pollution under certainty is fixed, $Ef_x < w$, implies that the use of pollution decreases This effect has already been noticed by several authors of the sixties and seventies showing the effect of uncertainty on input demand. They conclude that uncertainty reduces input demand because a risk premium is imposed on factors prices. See for example Sandmo, (1971); Holthausen, (1979); and MacMinn and Holtmann, (1983).

faced with greater economic uncertainty, always decreases pollution. However, there is no empirical evidence as to what the sign of θ is, even though the sign is likely positive.

The effects accounted by the first two effects in (11) reflect the impact of a MPS on $\tilde{f}$. The effect of the fall in its mean is given by the third term, $\eta\left(\frac{\delta}{\alpha}\right)$, in Eq. 11. This term is always positive given the concavity of $f(\cdot)$ and risk aversion. It shows the effect on pollution of a fall in $E\tilde{f}$. This term tends to increase pollution. Given the equivalence between the sign of the effect of a first order risk increase and a deterministic fall in income, this third term is akin to the effect of a non-stochastic reduction in f.

In summary, the first term on Eq. 11 -which can be called *a net precautionary effect*-, and the third term - which may be called a *complementary effect on production* -both tend to increase pollution, while the second effect – the *relative cross prudence in production*- is ambiguous.

Given its importance and novelty, we further explore cross-prudence. Note that by the definition of the elasticity of factor substitution, it is possible to write the following identity:

$$f_{xk} \equiv \frac{1}{\sigma}\frac{f_x f_k}{f}$$

Differentiating with respect to k, the following expression arises for θ:

$$\theta = -\frac{f_{xkk}f}{f_{xk}f_k} = \frac{E}{\alpha} + \left(1 - \frac{1}{\sigma}\right) + \frac{\delta}{\alpha} \qquad (13)$$

where E it is a measure of the curvature of f_x, which in macroeconomics is called super elasticity (elasticity of an elasticity) and defined as, $E = \sigma' k/\sigma$ or the elasticity with respect to capital of the elasticity of substitution (Kimball, 1995). Direct observation of Eq. 13 shows that, for the plausible case of non-decreasing elasticity of factor substitution $(i.e\ E \geq 0)$, θ is necessarily positive if the elasticity of factor substitution (σ) is greater than one.

To give more insights into Eq. 11 and 13, we will assume that $E = 0$, that is, we will specialize the analysis to a CES production function.

Proposition 3. *Let $f(k,x)$ be a CES production function with $\sigma \geq 1$. A Second Order Risk increase of $\tilde{k}$ (a mean preserving spread of $\tilde{k}$), raises pollution if $\eta > 1$ and if η is non increasing in f $(i.e. if\ \eta' \leq 0)$*

Proof. *Note that for a CES production function, using Eq. 13, it is possible to write Eq. 11 as:*

$$H_{kk} = \omega(k,x)[(\eta + \theta)(\eta - 1/\sigma) - \eta\eta'/a] \tag{14}$$

then,

$$Sign\ of\ H_{kk} = sign\ of\ [(\eta + \theta)(\eta - 1/\sigma) - \eta\eta'/a] \tag{14'}$$

where $a \equiv -u_{cc}/u_c$ is the coefficient of absolute risk aversion. Also, according to Eq. 14′, $H_{kk} > 0$ if $\eta > 1/\sigma$ and $\eta' \leq 0$. In addition, since $\sigma \geq 1$, then, $\eta > \frac{1}{\sigma}$. Therefore, $\eta > 1$ along with $\eta' \leq 0$ are sufficient conditions for H_{kk}> 0. QED.

If we restrict the set of utility classes analyzed to utility functions with a constant relative risk aversion (CRRA), the following corollary to Proposition 3 can be stablished:

Corollary 1. *Let $f(k,x)$ be a CES production function with $\sigma \geq 1$ and u a CRRA utility function. A Second Order Risk increase (a mean preserving spread of $\tilde{k}$), raises pollution if $P > 2$.*

Proof. *A CRRA utility function implies that, $\eta = P - 1$. Using the definition of θ in Eq. 13 and inserting the results in Eq. 14, yields the following result:*

$$H_{kk} = \omega(k,x)[(\eta + \theta)(\eta - 1/\sigma)] \tag{15}$$

with:

$$Sign\ of\ H_{kk} = Sign\ of\ (\eta + \theta)\left(P - 1 - \frac{1}{\sigma}\right). \tag{15'}$$

Given that $\sigma > 1$, a sufficient condition for $H_{kk} > 0$ is that $P > 2$. QED.

The intuition behind these results is clear. A second order risk increase in $\tilde{k}$ reduces the expected value of the marginal product of pollution below its marginal cost (below pollution price) if producers are cross-prudent; if the price of pollution does not change, firms would decrease pollution. But the expected supply price for pollution does change. If consumers are prudent, they would prefer to face the higher level of uncertainty with higher rather than lower income and will therefore be willing to accept higher levels of pollution. If relative prudence is high (for example because fear to put income too close to subsistence consumption), the price for accepting pollution will decrease more than the fall in the expected value of marginal product of pollution, inducing firms to increase pollution.

A high elasticity of factor substitution means that at times of uncertainty and stagnation increasing pollution is less costly. If σ is high, a small reduction in the price of pollution will be enough to induce firms to increase its emissions. The P coefficient on the other hand, shows how much environmental quality people are willing to give up for protecting income in times of economic uncertainty. Therefore, if P is large, the price of pollution that consumers are willing to accept will decrease more than if P is small.

Thus, the effect of economic uncertainty on pollution depends on two critical parameters, the elasticity of factor substitution and the coefficient of relative prudence. The greater the elasticity of factor substitution and the greater the coefficient of relative prudence, the more likely is that pollution will increase with a MPS in $\tilde{k}$.

Empirical evidence for $\boldsymbol{P}$ *and* $\boldsymbol{\sigma}$. Empirical findings suggest that both coefficients are quite large. For example, Fagereng, Guiso and Pistaferri (2017) have estimated that the coefficient of relative prudence is close to 2. Most empirical estimates of the relative risk aversion coefficient are quite high, often above 1 (e.g., Mehra and Prescott, 1985; Kocherlakota, 1996;

Barsky et. al. 1997; Cohen and Einav, 2007; Sydnor, 2010; Meyer and Meyer 2005). These findings are consistent with relative prudence above 2 as obtained by Fagereng et al.[10]

Also, it is often assumed that the elasticity of substitution between natural and man-made inputs is greater than 1 (Acemoglu, Aghion, Bursztyn, and Hemous; 2012), and empirical estimates of this elasticity are also quite high, often above 1 (Papageorgiou, Saam, and Schulte, 2017). Hence, if the CES specification for the production function and the CRRA one for the utility function are good approximations, the available empirical evidence would suggest that both a first and second order risk increase would contribute to raise pollution.

3. Implications

It is well known that increasing pollution may affect economic uncertainty (Tol, 2018; Weitzman, 2011; Archer, 2007). Research has shown that atmospheric CO2 accumulation causes climate change which increases the frequency and magnitude of climatic disasters such as hurricanes, droughts, and floods. In general, the variance of climate appears to dramatically increase as CO2 and other pollutants are accumulated in the atmosphere. This affects economic uncertainty making economic activity more unstable and by threatening capital stocks. Moreover, the rising levels of economic uncertainty caused by climate change is likely to be radically magnified in the future by the sequels of the recent pandemic. This makes the analysis of uncertainty even more relevant today than before.

Our model shows that it is more likely that this massive increase of economic uncertainty generated by the combined effects of climate change and the COVID-19 pandemic will lead to expanding levels of pollution especially once the short run productive impact of the pandemic

[10] Note that for CRRA the coefficient of relative prudence is $\eta + 1$.

subsides. Greater pollution, in turn, will worsen climate further, generating even more economic uncertainty and more pollution. This could lead to unstable equilibria eventually leading to economic and climatic collapse.

Also, the results stated so far imply that the upcoming technological automatization (which may cause a higher level of σ) implies that an increase of economic uncertainty increases pollution even more or that economic uncertainty increases pollution further for less prudent economies. This is a counterintuitive result and implies that economies that are less dependent on natural production factors may increase pollution even more in the face of greater economic uncertainty. This, in turn, may imply that economies are more exposed to unstable equilibrium. That is, greater automation may make the economy more vulnerable to cumulative instability; relatively small increases in economic uncertainty may have unsuspectedly high impact on the economy and the environment.

In addition, our model suggests that active fiscal policies that reduce economic instability may have an additional dividend by reducing the risks of environmental collapse. For example, countercyclical fiscal policy can mitigate the effects of climate change. Given this, implementing a carbon tax (or an increase of it) may be even a more powerful tool to reduce pollution if the revenues generated by this tax are used to finance countercyclical fiscal rules.

4. Conclusion

In this paper, we study the conditions under which greater uncertainty is likely to increase pollution. While in general this effect is quite complex involving a variety of contradictory mechanisms, we have shown that under certain common specifications often supported by empirical studies, pollution may rise when economic uncertainty increases. In general, the value of the technical elasticity of substitution between pollution and other conventional factors of production as well as the coefficient of risk aversion and prudence coefficient shape the relationship between economic uncertainty and pollution.

Using parametric examples for the utility function and the production function, we show that the conditions for which pollution increases in the face of greater economic uncertainty are likely to hold. This result suggests that when uncertainty is caused by worsening environmental conditions (e.g., climate change) a vicious cycle of worsening environment-greater uncertainty-even greater uncertainty and so forth may occur. This is probably the most important message of this paper.

The existence of a vicious cycle as the one described in this paper are largely triggered by the fact that countries are assumed to optimize their pollution policies considering only their own welfare. The optimal response to greater economic uncertainty at the country level is likely to allow more pollution emissions in times of uncertainty. If uncertainty is caused by increasing pollution at the world level (e.g., anthropogenic CO2 atmospheric concentrations), then the loosening of emission controls by all or most countries may induce a cumulative process leading to climatic collapse.

This result is not merely one more example of the tragedy of the commons caused by uncontrolled externalities. In fact, it is much worse; in the regular tragedy of the commons case depletion of the common resource reduces the incentives for its use. A lake been depleted of its fish stock by over exploitation becomes less attractive to fishermen as the catch becomes progressively more costly to extract and therefore its exploitation becomes less intense, which in many cases allows to prevent complete extinction.

By contrast, in the case we analyze in this paper, which we may call a *perverse tragedy of the commons*, over exploitation of the atmosphere though increasing pollution, does not reduce the incentives to continue its exploitation. On the contrary, through the increasing risk and uncertain mechanism, it generates incentives to ever more intense exploitation of the atmosphere through expanding pollution. If the commons problem is not solved through international coordination and cooperation this phenomenon may lead to the complete collapse of the atmospheric resource and humanity as well.

These results give even more urgency to the need of increasing cooperation among countries by coordinating policies to control climate change. Policy cooperation and coordination is the only way of reducing the danger of triggering the unstable processes emphasized in this paper, which could hasten even further climatic and eventually economic collapse.

Appendix A: Proof of propositions 1

To prove Propositions 1 (and Proposition 2 as well), we develop some measures of change in Risk based on Chiu and Eeckhoudt (2010). Assume that the random variable $\tilde{k}$ is positive and bounded above with probability one and denote its cumulative distribution function as $F_{\tilde{k}}$. For a distribution function $F_{\tilde{k}}(z)$, define $F_{\tilde{k}}^{(1)}(z) = F_{\tilde{k}}(z)$ and,

$F_{\tilde{k}}^{(n+1)}(z) = \int_0^x F_{\tilde{k}}^{(n)}(y)dy$ for all $z > 0$ and all $n \in \{1,2, \ldots.\}$

The definition of Nth degree risk increase according to Ekern (1980) is presented below.

Definition 1

$\tilde{k}_1$is a Nth degree increase of $\tilde{k}_0$ if $F^{(N)}_{\tilde{k}_0}(z) \leq F^{(N)}_{\tilde{k}_1}(z)$ for all $z > 0$ where the inequality is strict for some z and $F^{(n)}_{\tilde{k}_0}(\infty) = F^{(n)}_{\tilde{k}_1}(\infty)$ for $n = 2, \dots., N$

$F^{(n)}_{\tilde{k}_0}(\infty) = F^{(n)}_{\tilde{k}_1}(\infty)$ for $n = 2, \dots., N$ means that the first $(N-1)$ moments of $\tilde{k}_0$ and $\tilde{k}_1$ are equal. Therefore, $\tilde{k}_1$ as a first-degree risk increase of $\tilde{k}_0$ is the same as $\tilde{k}_0$ dominating $\tilde{k}_1$ by first-degree stochastic dominance (FSD). Some of the increases in risk fulfilling the above definition are a mean preserving spread (Rothschild and Stiglitz, 1970), which is a second-degree risk increase; a downside risk increase (Menezes et al., 1980), which is a third-degree increase in risk; and an increase in outer risk (Menezes and Wang, 2005), which is a fourth-degree increase in risk (see Chiu and Eeckhoudt, 2010). Higher-order increases in risk are characterized by Eeckhoudt and Schlesinger (2006).

In the expected utility framework, the properties of these concepts are characterized by the following lemma (Chiu and Eeckhoudt, 2010)

Lemma 1

(i) *For $\tilde{k}_1$ being a Nth degree risk increase over $\tilde{k}_0$, $Eu(\tilde{k}_1) \leq (\geq) Eu(\tilde{k}_0)$ if and only if $(-1)^N u^{(N)} \leq (\geq)\ 0$*

We start by considering the simplest case of a first-degree increase in risk (equivalent to the concept of first-order stochastic dominance, FSD). Consider a first-degree increase in risk from a non-random k_0 to a random $\tilde{k}$ where $E(\tilde{k}) = k_0$.

For a random $\tilde{k}$ the problem to solve by social planner is $Max\ Eu\big(f(\tilde{k}, x), 1 - x\big)$, with first order condition given by $E\big[u_c\big(f(\tilde{k}, x^*), 1 - x^*\big) f_x(\tilde{k}, x^*) - u_q\big(f(\tilde{k}, x^*), 1 - x^*\big)\big] = 0$

It follows that the optimal level of pollution will increase (decrease) whenever

$$E\left[u_c\left(f\left(\tilde{k}, x_0\right), 1-x_0\right) f_x(\tilde{k}, x_0) - u_q\left(f\left(\tilde{k}, x_0\right), 1-x_0\right)\right]$$
$$- E\left[u_c(f(k_0, x_0), 1-x_0) f_x(k_0, x_0) - u_q(f(k_0, x_0), 1-x_0)\right] > (<)\ 0$$

Define $H(k, x_0) = u_c(f(k, x_0), 1-x_0) f_x(k, x_0) - u_q(f(k, x_0), 1-x_0)$

pollution increase (decrease) whenever

$EH\left(\tilde{k}, x_0\right) > (<) EH(k_0, x_0)$

By Lemma 1, $EH\left(\tilde{k}, x_0\right) > (<) EH(k_0, x_0)$, for $\tilde{k}$ being a 1st degree risk increase of k_0 if and only if $(-1)H_k(k, x^*) > (<) 0$, and thus, if and only if

$$H_k(k, x_0) = f_{xk}u_c + f_x u_{cc} f_k - u_{qc} f_k < (>)\ 0 \qquad \text{(A1)}$$

which is equal to Eq 6 in the main text.

Note Eq. A1 is the same condition for the effect on pollution of a deterministic reduction in k,

This can be shown, totally differentiating Eq. (3) with respect to k and x shows that pollution increases (decreases) if,

$$\frac{\partial \mathrm{x}}{\partial \mathrm{k}} = -\frac{H_k}{H_x} = -\frac{f_{xk}u_c + f_x u_{cc} f_k - u_{qc} f_k}{H_x} < (>) 0$$

where $H_x < 0$ for the second order condition to hold.

This equivalent to $H_k(k, x) = f_{xk}u_c + f_x u_{cc} f_k - u_{qc} f_k < (>)\ 0$ which is equal to Eq. A1

Eq. A1 can also be expressed as:

$$H_k(k, x_0) = \frac{f_x f_k u_f}{f}\left[\frac{f_{xk} f}{f_x f_k} + \frac{u_{cc}}{u_c} c - \frac{u_{cq}}{f_x u_c} c\right] \tag{A2}$$

defining $\sigma \equiv \frac{f_x f_k}{f_{xk} f}$ as the elasticity of factor substitution, $\eta \equiv -\frac{u_{cc}}{u_c} c$, as the coefficient of relative risk aversion, and $\gamma \equiv -\frac{u_{qc}}{u_q} c$, the coefficient of relative correlation aversion, and remembering that $f_x = u_q/u_c$, Eq A2 reduces to Eq. 7 in the text.

From there Proposition 1 follows. *QED*

Appendix B: Proof of Proposition 2 assuming additive preferences

Consider now a second-degree increase in risk (mean preserving increase in risk) from a non-random k_0 to a random $\tilde{k}$ where $E(\tilde{k}) = k_0$. It follows, now that the optimal level of pollution will increase (decrease) whenever

$$E\left[u_c\left(f\left(\tilde{k}, x_0\right), 1 - x_0\right) f_x(\tilde{k}, x_0) - u_q\left(f(\tilde{k}, x_0), 1 - x_0\right)\right] - E\left[u_c(f(k_0, x_0), 1 - x_0) f_x(k_0, x_0) - u_q(f(k_0, x_0), 1 - x_0)\right] > (<)\, 0$$

pollution increase (decrease) whenever

$$EH\left(\tilde{k}, x_0\right) > (<) EH(k_0, x_0)$$

By Lemma 1, $EH\left(\tilde{k}, x_0\right) > (<) EH(k_0, x_0)$, for $\tilde{k}$ being a 2^{nd} degree risk increase of k_0 if and only if $(-1)^2 H_{kk}(k, x^*) > (<) 0$, and thus, if and only if

$$H_{kk}(k,x_0) = 2f_{xk}u_{cc}f_k + f_x u_{ccc}f_k^2 + f_{xkk}u_c + f_x u_{cc}f_{kk} - u_{qcc}f_k^2 - u_{qc}f_{kk} > (<)0 \quad \text{(A3)}$$

which is Eq. 10 under additive preferences ($U_{qc} = U_{qcc} = 0$). Assuming additive preferences, Eq. A3 can be expressed as

$$H_{kk}(k,x_0) = -\frac{u_{cc}f_k^2 f_x}{f}\left(P - \frac{2}{\sigma}\right) + f_{xkk}u_c + f_x u_{cc}f_{kk} \quad \text{(A4)}$$

where $P \equiv -u_{ccc}c/u_{cc}$ is the coefficient of relative prudence. Further developing Eq. (A4)

$$H_{kk}(k,x_0) = -\frac{U_{cc}f_k^2 f_x}{f}\left[\left(P - \frac{2}{\sigma}\right) - \frac{f_{xkk}U_c}{U_{cc}f_k^2 f_x}f - \frac{f_x U_{cc}f_{kk}f}{U_{cc}f_k^2 f_x}\right]$$

$$= -\frac{U_{cc}f_k^2 f_x}{f}\left[\left(P - \frac{2}{\sigma}\right) + \frac{1}{\eta}\frac{f_{xkk}f}{f_{xk}f_k}\frac{f_{xk}f}{f_x f_k} - \frac{\frac{kf_{kk}}{f_k}}{\frac{kf_k}{f}}\right] \quad \text{(A5)}$$

Defining $\delta \equiv -kf_{kk}/f_k > 0$, $\alpha \equiv kf_k/f$, and, $\theta \equiv -f_{xkk}f/f_{xk}f_k$, the coefficient of relative cross-prudence in production, and remembering that $\eta \equiv -u_{cc}c/u_c$ is the coefficient of relative risk aversion and $\sigma = f_x f_x/f_{xk} f$, the elasticity of factor substitution, Eq. A5 can finally be expressed as

$$H_{kk}(k,x_0) = \frac{u_c {f_k}^2 f_x}{f^2}\left[\eta\left(P - \frac{2}{\sigma}\right) - \frac{\theta}{\sigma} + \eta\frac{\delta}{\alpha}\right]$$

which is Eq. 11 in the main text. *QED*.

Appendix C: Proof of Proposition 2 for non-additive preferences

For non-additive preferences, it is possible to write Eq. A3 as:

$$H_{kk} = \frac{u_c {f_k}^2 f_x}{f^2}\left[\eta\left(P - \frac{2}{\sigma}\right) - \frac{\theta}{\sigma} + \eta\frac{\delta}{\alpha}\right] - u_{qcc}f_k^2 - u_{qc}f_{kk} > (<)0)$$

Thus, under concavity of the production function, one additional effect of second-degree increase in risk to increase (decrease) pollution exist whenever:

$$u_{qcc} < (>)0 \text{ and } u_{qc} > (<)0 \quad \text{(A6)}$$

The first term in Eq. A6, u_{qcc}, is an indicator of cross prudence in environment, and works toward increased (decreased) pollution if $u_{qcc} < (>) \; 0$. The second term u_{qc} is positive if the environment is a normal good and negative if it is an inferior good. In this case increasing uncertainty in $\tilde{k}$ decreases pollution if people are imprudent in environment ($u_{qcc} < 0$) and they will prefer to face the higher uncertainty with low rather than high environmental quality. On the other hand, if $f_{kk} < 0$, expected income decreases and if the environment is a normal good, people will demand less environmental quality. This is the last term in Eq. A5. $U_{qcc} < 0$ and $u_{qc} > 0$ work toward higher levels of pollution.

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