Collusion sustainability with optimal punishments and detection lags, with an application to a Cournot game

Sostenimiento de colusión con castigos óptimos y retrasos en la detección, con una aplicación al juego de Cournot

Aitor Ciarreta*
Carlos Gutiérrez-Hita**

Abstract

In this paper we characterize optimal punishments with detection lags when the market consists of n oligopolistic firms that compete à la Cournot. It is shown how in the presence of detection lags optimal punishments fail to restore cooperation as long as the number of lags increases. Moreover, collusion sustainability is difficult to achieve also if the number of firms is low.

Key words: *Optimal punishments, detection lags, collusion sustainability.*

JEL Classification: *C73, D43.*

Resumen

En este artículo caracterizamos castigos óptimos con retrasos en la detección en un mercado oligopolístico con n empresas que compiten à la Cournot. Se demuestra que en presencia de retrasos en la detección los castigos óptimos no logran restaurar la cooperación a medida que el número de retrasos aumenta. Además, la colusión es también más difícil de sostener si el número de empresas es bajo.

---

* Departamento de Fundamentos del Análisis Económico II, Universidad del País Vasco, Bilbao, España.
Aitor Ciarreta thanks Ministerio de Economía y Competitividad of Spain (ECO2012-35820) and Gobierno Vasco (DEUI, IT-313-07) for their financial support. Carlos Gutiérrez-Hita acknowledges support from the Spanish Department of Science and Technology (through research project MTM2011-23205), and Basque Government (through research project DEUI, IT-313-07). The usual disclaimers applied.
** Depto. Estudios Económicos y Financieros, Universitas Miguel Hernández, Elche (Alicante), Spain.
(Corresponding author). Mailing address: Av/Universidad s/n, 03202, Elche (Alicante), Spain. Tel.: +34 966658869; fax: +34 966 658564. E-mail address: cgutierrez@umh.es; cgutierrezhi@uoc.edu.

Palabras clave: *Castigos óptimos, retardos en la detección, sostenimiento de la colusión.*

Clasificación JEL: *C73, D43.*

1. Introduction

Strategic interaction plays an important role to determine the sustainability of cartel agreements. Early works by Stigler (1964) and Selten (1973) find a monotonic decreasing relationship between the likelihood of collusion and the number of participant firms. The former focuses on cartel maintenance whereas the latter does it on cartel formation. Werden and Baumann (1986) extend the analysis in both dimensions to find that the relationship between the likelihood of collusion and the number firms is not necessarily monotonic once fines and damages are included. These models rely on providing conditions for the existence of self-sustained cartels in static framework.

Friedman (1971) stressed the importance of time for collusion sustainability under trigger strategies profile. Abreu (1986, 1988) also characterized optimal punishments in Supergames. He shows that a symmetric optimal penal code yields the lowest critical discount factor such that collusion can be sustained. Moreover, Abreu *et al.* (1986) and Fudenberg and Maskin (1986) characterize folk theorems for infinitely repeated games with discounting.

In this paper we explore the impact of detection lags in collusion sustainability under Abreu's optimal punishment penal code. Detection lags are almost impossible if there is perfect observation.[1] Therefore, the existence of imperfect observation of secret deviations implies that detection lags are not certain but random. Green and Porter (1984) build a model of cooperation with imperfect cooperation where firms monitor price. Hence, in their model demand fluctuations imply that only if the price falls below a certain level triggers competition. That is, firms are unable to perfectly distinguish demand shifts with secret violations of the agreed level of production by rivals.

We want to emphasize the importance that detection lags have in collusion sustainability. It is assumed that cheating on an implicit agreement can be kept secret for a certain time period because of the inability to retaliate in one period. In fact, when a firm cheats from the collusive agreement is possible that the rest of cartel partners take some periods in detecting that deviation. This can be possible for a number of reasons, for instance, the number of firms in the market. When the market is composed by a high number of competitors a deviation is difficult to detect as long as the effect of a variation in firm's strategy has a little impact on price formation. Indeed, small differences in final price could be the result of minor demand shocks in the short run. Another example is when market transactions take place out of the market place, directly between a buyer and a seller. This is the case of certain input markets where wholesalers directly

1 We would like to thank an anonymous reviewer for suggesting this discussion on detection lags in the context of the paper.

negotiate with downstream firms. In the case that wholesalers have an agreement to restrict input supply or to sell it at higher prices a deviation is difficult to detect for the rest of cartel members because transactions take place *bis-a-bis*.

We revisit Friedman's (1971) work under optimal punishment instead of trigger strategies to characterize the effect of detection lags in collusive agreements in an *n*-firm oligopoly. We reach two interesting insights. First, optimal punishments fail to restore cooperation for a sufficiently large detection lag. For a given number of firms, as long as the number of periods to detect a deviation increases it is necessary a higher discount factor to sustain collusion (*i.e.*, firms have to be more patient). Secondly, it is found an inverse relationship between the amount of detection lags and the maximum number of firms compatible with collusion sustainability.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the main results. Section 3 concludes.

2. The Model

Consider an infinitely repeated *n*-firm game ($n \geq 2$) with discounting δ, $0 < \delta < 1$. Call such a game $G^l(n,\infty,\delta)$ where the superscript denotes that deviation is detected after l period(s); that is, we assume that an individual deviation from any collusive agreement is detected by the rest n–1 firms after l period(s), for $l = 1,2,...,T$. Firms agree upon the following penal code: if a deviation occurred at any given time τ it is detected at the end of the period $\tau + 1$, then firms adopt the punishment strategy symmetrically at time $\tau + l + 1$; finally, they restore cooperation from period $\tau + l + 2$ onwards. If, otherwise, any firm at time $\tau + l + 1$ does not join the penal code and decides not to retaliate, the punishment phase continues until the same action is taken by the n firms.

We call the best collusive strategy $s^* = (s_1^*,...,s_i^*,...,s_n^*)$, and the optimal symmetric punishment strategy $s^c = (s_1^c,...,s_i^c,...,s_n^c)$. Call $\pi(s^*)$ the best collusive profits, $\pi^d(s^*)$ the profits that would arise in case of deviation from the best collusive path, $\pi(s^c)$ are the profits during the punishment phase, and $\pi^d(s^c)$ are optimal deviation profits from the punishment phase.

In the repeated game with optimal punishment and discounting, we need two inequalities to find the range of the discount factor such that collusion is sustainable; (1) absence of private incentives to deviate, and (2) absence of private incentives to deviate from the punishment phase. Regarding inequality (1), there are no private incentives to deviate from the collusive path if

$$(1) \qquad \sum_{t=0}^{\infty} \delta^t \pi(s^*) \geq \sum_{t=0}^{l} \delta^t \pi^d(s^*) + \delta^{i+1}\pi(s^c) + \sum_{t=l+2}^{\infty} \delta^t \pi(s^*)$$

That is, the present discounted payoffs from collusion must be larger than the sum of the present discounted payoffs from deviation, plus payoffs during the one-period punishment phase plus the payoffs from returning to the collusive path after $l + 2$ periods. Regarding inequality (2), colluding firms do not have private incentives to deviate from the optimal penal code if

(2) $$\pi(s^c)+\sum_{t=1}^{\infty}\delta^t\pi(s^*)\geq\sum_{t=0}^{l}\delta^t\pi^d(s^*)+\delta^{i+1}\pi(s^c)+\sum_{t=l+2}^{\infty}\delta^t\pi(s^*)$$

That is, the payoffs that can be obtained during the punishment phase must be at least as large as the payoffs each firm can obtain by deviating from the punishment phase. Note that if we take $l=0$ then it is the same set of inequalities that can be found in Abreu (1986, 1988).

2.1. Collusion sustainability

In this subsection we provide conditions such that collusion can be sustained in the infinitely repeated game with discounting. Proposition 1 below characterizes the optimal penal code when l detection lags are included.

Proposition 1. Let (s^*,s^c) be the optimal stick-and-carrot punishment with detection lags. Then the following is hold,

(3) $$\sum_{t=0}^{l}\delta^t\left[\pi^d(s^*)-\pi(s^*)\right]\leq\delta^{l+1}\left[\pi(s^*)-\pi(s^c)\right]$$

(4) $$\sum_{t=0}^{l}\delta^t\left[\pi^d(s^c)-\pi(s^c)\right]=\sum_{t=1}^{l+1}\delta^t\left[\pi(s^*)-\pi(s^c)\right]$$

where l is the number of detection lags.

Proof: First, consider inequality (1). Thus, $\sum_{t=0}^{\infty}\delta^t\pi(s^*)$ can be rewritten as $\sum_{t=0}^{l}\delta^t\pi(s^*)+\sum_{t=l+1}^{\infty}\delta^t\pi(s^*)$. Therefore (1) can be expressed as

$$\sum_{t=0}^{l}\delta^t\left[\pi^d(s^*)-\pi(s^*)\right]\leq\sum_{t=l+1}^{\infty}\delta^t\pi(s^*)-\sum_{t=l+2}^{\infty}\delta^t\pi(s^*)-\delta^{l+1}\pi(s^c)$$

where the LHS is nothing but the LHS of equation (3). Besides, the difference $\sum_{t=l+1}^{\infty}\delta^t\pi(s^*)-\sum_{t=l+2}^{\infty}\delta^t\pi(s^*)$ is equal to $\delta^{l+1}\pi(s^*)$, therefore we obtain the RHS of (3). Second, consider inequality (2). Thus, by subtracting $\sum_{t=1}^{l+1}\delta^t\pi(s^*)$ on both sides of (2) it is obtained

$$\sum_{t=0}^{l}\delta^t\pi^d(s^c)-\sum_{t=1}^{l+1}\delta^t\pi(s^c)-\pi(s^c)+\delta^{l+1}\pi(s^c)=\sum_{t=1}^{\infty}\delta^t\pi(s^*)-\sum_{t=l+2}^{\infty}\delta^t\pi(s^*)-\sum_{t=1}^{l+1}\delta^t\pi(s^c)$$

The last three terms on the LHS are just $\sum_{t=0}^{l}\delta^t\pi(s^c)$. Hence, we get the LHS of (4). Finally, take $\sum_{t=1}^{\infty}\delta^t\pi(s^*)-\sum_{t=l+2}^{\infty}\delta^t\pi(s^*)$ on the RHS which is $\sum_{t=1}^{l+1}\delta^t\pi(s^*)$. Therefore, we get the RHS of (4). This completes the proof.

Equation (3) is the no-defection condition whereas Equation (4) requires that each firm does not deviate from the punishment path after deviation is detected in the game with l detection lags. If that was the case, punishment would follow until all firms agree to go along. If we assume one detection lag, $l = 0$, and the assumption that the best collusive outcome is achieved, Theorem 15 of Abreu (1986, p. 203) is obtained.

2.2. Quantity competition and detection lags

Let us consider the case of quantity competition in order to illustrate the predictions of Proposition 1. Assume market demand is given by $p(Q)=1-Q$ where $Q=\sum_{i=1}^{n}q_i$. Marginal costs are zero for each firm.[2] As firms engage in quantity competition a strategy s_i is just the choice of the optimal quantity under collusion, q_i^*, deviation, q_i^d, and the punishment phase, q_i^c. By using Proposition 1 and the best response function $q_i(q_{-i}^c)=[1-c-(n-1)q_{-i}^c]/2$ where q_{-i}^c is the punishment strategy of the remaining n–1 firms, equations (3) and (4) can be expressed as,

$$\sum_{t=0}^{l}\delta^t\left[\frac{(n+1)}{16n^2}-\frac{1}{4n}\right]=\delta^{t+1}\left[\frac{1}{4n}-(1-nq_i^c)q_i^c\right],$$

$$\sum_{t=0}^{l}\delta^t\left[(1-q_i(q_{-i}^c)-nq_{-i}^c)q_i(q_{-i}^c)-(1-nq_i^c)q_i^c\right]=\sum_{t=1}^{l+1}\delta^t\left[\frac{1}{4n}-(1-nq_i^c)q_i^c\right].$$

In order to obtain the main results we proceed as follows: (i) for a given number of detection lags find the maximum number of firms compatible with a discount factor $\delta < 1$, and (ii) for a given number of firms find the number of detection lags which yields $\delta < 1$. Corollary 1 summarizes the results.

Corollary 1. Consider the game $G^l(n,\infty,\delta)$ with optimal punishments à la Abreu. Thus, under Cournot competition collusion is sustainable accordingly the following trade-off between number of lags and number of firms,

[2] This assumption allows for negative prices during the punishment phase but does not affect qualitative results. This is done for ease of exposition.

TABLE 1
VALUES, LAGS AND NUMBER OF FIRMS.

# lags	1	2	4	7	14	16	17	270
2 firms	0.461	0.564	0.682	0.773	0.864	0.878	0.885	0.999
3 firms	0.528	0.637	0.757	0.846	0.931	0.943		
4 firms	0.600	0.714	0.835	0.920	0.994			
5 firms	0.673	0.790	0.911	0.990				
6 firms	0.746	0.866	0.984					
7 firms	0.818	0.939						
8 firms	0.890							
9 firms	0.961							

Table 1 reports simulations of the minimum value of the discount factor δ such that collusion is sustainable for each combination of detection lag and number of firms.[3] For example, with seven detection lags five firms are able to sustain collusion over time if and only if $\delta \in (0.99,1)$. It is shown that there is an inverse relationship between the amount of detection lags and the number of firms compatible with collusion over time. Moreover, for a given number of firms, as the amount of detection lag increases a higher δ is needed to maintain the collusive agreement.

3. Concluding Remarks

In this paper we introduce detection lags in an infinitely repeated game with discounting. First, it is shown that under quantity competition, given a number of firms, Abreu's stick-and-carrot punishment fails to restore cooperation for a sufficient large detection lag. This result calls for attention to detect collusion in markets characterized by infrequent interaction or imperfect information. Further research calls for assuming randomness in the number of detection lags. Second, we find that as long as the number of lags increase collusion is more difficult to sustain also if the number of firms is low.

References

Abreu, D.J. (1986). "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*. Vol. 39; 191-225.

Abreu, D.J. (1988). "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica*. Vol. 56; 383-396.

Abreu, D.J.; Pearce, D. and Stacchetti, E. (1986). "Optimal Cartel Equilibria with Imperfect Monitoring", *Journal of Economic Theory*. Vol. 39; 251-269.

[3] Upon request authors send the program which provides the results reported. Calculations have been run with the computer package Mathematica® by Wolfran Research.

Friedman, J.W. (1971). “A Non-cooperative Equilibrium for Supergames”, *Review of Economic Studies*. Vol. 28; 1-12.

Fundenberg, D. and Maskin, E. (1986). “The Folk Theorem in Repeated Games with Discounting or with Incomplete Information”, *Econometrica*. Vol. 54; 533-554.

Green, E. and Porter, R. (1984). “Noncooperative Collusion Under Imperfect Price Information”, *Econometrica*. Vol. 52; 87-100.

Selten, R. (1973). “A Simple Model of Imperfect Competition, when 4 Are Few and 6 Are Many”, *International Journal of Game Theory*. Vol. 2 (3); 141-201.

Stigler, G. (1964). “A Theory of Oligopoly”, *Journal of Political Economy*, Vol. 72 (1); 44-61.

Werden, G.J. and Baumann, M.G. (1986). “A Simple Model of Imperfect Competition in which Four are Few but Three Are Not”, *The Journal of Industrial Economics*. Vol. 34 (3); 331-335.