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# SOURCES OF UNCERTAINTY FOR CONDUCTING MONETARY POLICY IN CHILE

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# SOURCES OF UNCERTAINTY FOR CONDUCTING MONETARY POLICY IN CHILE*

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**Abstract**

This paper analyzes the quantitative relevance of additive, multiplicative and data uncertainty in the implementation of Chile's monetary policy. For the analysis of data uncertainty we focus on the uncertainty associated with the estimation of the output gap using real-time data and various well-known methods to estimate the output trend. We found that the revisions of the output gap are important and persistent and that the unobserved components method shows a better performance with real-time data than other more usual ones, like the HP filter. In the case of additive and multiplicative uncertainties we estimate the equations that govern the behavior of the economy with time-varying parameters and with state-dependent variances in the shocks of the model. This allows us to analyze the contribution of these two types of uncertainties on the total uncertainty. We found that additive uncertainty is the most relevant to explain total uncertainty and that shocks to the model are state-dependent.

*JEL Classification: E32, E58, E59, C32.*

*Key words: real-time data, output gap, time-varying parameter models, models with regime changes, forward-looking models.*

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# 1. Introduction

It is widely accepted that monetary policy is made in an environment of substantial uncertainty. This has increased considerably the interest of academic researchers to formally demonstrate the implications of uncertainty, as well as the ways in which central banks can deal with uncertainty. The theoretical literature on uncertainty distinguishes between three types of uncertainty: *Additive uncertainty*, which refers to the lack of knowledge of the central banks regarding the future shocks faced by the economy; *Multiplicative uncertainty*, which represents the lack of knowledge, or the erroneous knowledge, of one or more parameters of the behavioral model of the economy; and *Data uncertainty*, which is associated to the fact that the information used by the central bank at the time policy decisions are made could either be incorrect or could incompletely reflect the actual state of the economy. The objective of this paper is to review the quantitative relevance of these three types of uncertainty in the Central Bank of Chile's monetary policy. The paper is divided into two parts: the first covers the problem of data uncertainty and focuses on the output gap estimates for the *full-fledged inflation targeting* period (1999 onward); the second centers on additive and multiplicative uncertainty for the period 1990 - 2006 but places a special emphasis on the period subsequent to 1999.

In the analysis of data uncertainty we focus on the output gap given its importance in forecasting inflation and given that preliminary figures for real output (real-time data), which are revised several times, are available when monetary decisions are made. Also, the estimation of the output trend (part of the output gap) depends on statistical filters applied to output series, which contain these preliminary figures. For our exercise, we use various well-known univariate filters: the Hodrick-Prescott (HP), the Baxter-King (BK), the Christiano-Fitzgerald (CF), the quadratic trend and the Clark method based on the unobserved components model. To analyze their reliability and statistical accuracy with real-time data we follow the methodology proposed by Orphanides and van Norden (1999). We find that revisions of the output gap in the case of Chile are important and persistent, and that correlations between the final data output gap and the real-time data output gap are relatively low. Nonetheless, the Clark method produces the best results, implying that caution and judgment should be taken when evaluating the business cycle with real-time data, and that using popular filters, like HP, could be misleading.

To evaluate the empirical importance of additive and multiplicative uncertainty we use the methodology proposed by Zhang and Semmler (2005). In particular, we estimate behavioral equations for the Chilean economy with time-varying parameters and shocks with state-dependent variance (two states), which follow a first order Markov process. To estimate behavioral equations we use a slightly modified version of the *forward-looking* specification of Svensson (2000) and Al-Eyd and Karasulu (2006) for the equations that govern the behavior of a small open economy – the aggregate demand, the Phillips curve, and the real uncovered interest parity condition. Additionally, we use a technique from Kim (1993) to decompose total uncertainty, measured using the conditional variance of the forecast error, into two components: that associated to multiplicative uncertainty and that associated to additive uncertainty. We find that for all the behavioral equations of the economy, the uncertainty of shocks (additive uncertainty) has been the most important in explaining total uncertainty. Moreover, the estimations support the hypotheses of state-dependent variances and that these states could be considered as periods of high and low volatility in the shocks. Also, total uncertainty of both the output gap and the inflation rate has declined over time and the period of greater stability coincides with the establishment of the *full-fledge inflation targeting* framework for the conduct of monetary policy.[1]

The paper is organized as follows. In section 2 we present a literature review on the types of uncertainty faced by central banks, its implications for the conduct of monetary policy and the way in which they have been typically modeled empirically. In section 3 we analyze the quantitative relevance of data uncertainty, focusing on the output gap estimates. In section 4 we take on the importance of additive and multiplicative uncertainty in the models typically used to study the effect of monetary policy. Finally, concluding remarks are presented in section 5.

# 2. Monetary policy and uncertainty

In the last few years, there has been a considerable increase in the interest of academic researchers to demonstrate formally the ways in which central banks can deal with uncertainty (Schellekens, 2002; Feldstein, 2003). Some papers have studied the distinct types of uncertainty faced by central banks, which have introduced important challenges in the modeling of monetary policy, and its implications on the behavior of the monetary authority. Such is the case of Isard et.al. (1999), Martin and Salmon (1999), Svensson (1999), Wieland (2000), Meyer et.al. (2001), Tetlow and von zur Muehlen (2001), Gianoni (2002), Orphanides and Williams (2002), and Soderstrom (2002). Other papers have proposed different strategies that can be used to deal with uncertainty, namely robust monetary policy rules and learning mechanisms, among others. See for example Craine (1979), Holly and Hughes Hallett (1989), Basar and Salomon (1990), Bertocchi and Spagat (1993), Balvers and Cosimano (1994), Sargent (1998), Onatski and Stock (2000), and Wieland (2000).

Feldstein (2003) argues that central banks typically face four types of uncertainty: uncertainty about the current and future states of the economy, uncertainty about how the economy operates, uncertainty of individuals about their personal futures, and uncertainty about the impact of potential future monetary

[1] It is important to mention that this period also coincides with the establishment of the structural surplus rule for the conduct of fiscal policy and with a generally highly stable international context.

policies. However, the most common classification defines three types of uncertainty: additive uncertainty, multiplicative uncertainty and data uncertainty.[2] *Additive uncertainty* represents the component of a forecast error associated to the outcome of an exogenous variable in the system (the regression model error). This type of uncertainty captures the lack of knowledge of central banks regarding the future shocks faced by the economy (Zhang and Semmler, 2005; Grauwe, 2006). *Multiplicative (or parameter) uncertainty*, on the other hand, represents the lack of knowledge, or the erroneous knowledge of one or more parameters of the behavioral model of the economy (and its agents). Hall et.al. (1999) claims that this type of uncertainty can occur for several reasons: the stochastic nature of the parameters, the measurement errors in the data utilized to estimate the model, and structural changes. The distinction between additive uncertainty and multiplicative uncertainty is based on the assumption that the true behavioral model of the economy is known. The limitation of this assumption is that total uncertainty, which could also result from misspecification of the model, is underestimated and, therefore, the results of any efforts to quantify this uncertainty using a particular specification of the behavioral model of the economy should be taken with caution.[3] Finally, *data uncertainty* is associated to the fact that the information used by the central bank at the time policy decisions are made could either be incorrect or could incompletely reflect the actual state of the economy (Orphanides and van Norden, 1999). Rudebush (2001) says that when these types of uncertainty are combined they weigh heavily on policy decision-makers. Having no knowledge of the actual state of the economy (be it due to uncertainty in the data or in the behavior of the economy) forces policy-makers to base their decisions on expected outcomes, which when more than one exists, could generate dilemmas in the adoption of an adequate policy (i.e. a more aggressive or passive reaction by the Central Bank).

Phillips (1954) and Theil (1964) were the first to introduce the idea of additive uncertainty, and their contributions have led to the expansion of the literature in this area. Phillips (1954), in studying whether stabilization policy recommendations of the simple models based on multipliers are appropriate or not and under what conditions this occurs, showed that in a system that is *automatically regulated* (with flexible prices and interest rates), monetary policy could be a suitable instrument to stabilize the economy, or at least to maintain the economic system close to its desired values. Monetary policy should also be able to deal with shocks, except the most severe ones. Theil (1964), assuming that the policy-makers choose their

[2] Another type of uncertainty also considered in the literature, but not analyzed in this paper, is uncertainty about the probability distributions over possible events known as Knightian uncertainty.

[3] Even though part of the existing literature defines multiplicative uncertainty as the lack of knowledge of the parameters and of the model, the distinction between both is important from a practical point of view. If the distinction is not made, it is not possible to separate the concepts of additive and multiplicative uncertainty given that any specification error affects both the regression error and the magnitude of the parameters (bias).

policy by maximizing a quadratic expected utility, expanded on the idea of Phillips (1954) as he found that in a world where there is only uncertainty in shocks, policy-makers could conduct their policy as if there were total certainty regarding the possible outcomes of the economy. This result is known as *certainty-equivalence* and has important implications for monetary policy.

Of course, there was a high degree of confidence in econometric modeling, such that in the estimation of structural models any error could be eliminated, except that associated to additive uncertainty. However, the principle of certainty-equivalence is valid only under certain conditions, particularly, in a linear quadratic world; therefore, policy implications could be different depending on the assumptions adopted regarding the behavior of the Central Bank (i.e., its loss function). As a matter of fact, Walsh (2003) found that optimal monetary policy rules, derived from a quadratic loss function for the Central Bank, are robust under this type of uncertainty and do not require that the monetary authority change its rule in the presence of shocks. However, simple Taylor reaction functions can generate important increases in the Central Bank's loss function depending on whether, based on particular situations, they require changes in the Central Bank's behavior. Additionally, Sack (2000), in estimating and simulating a VAR model for the US economy under different assumptions, found that if the only source of uncertainty is additive, the FED should show a more aggressive behavior than what it really shows in practice and argues that there are other types of uncertainty such as multiplicative, which generate greater gradualism in its monetary policy.

In a paper concerning linear decision rules for stabilization and growth, Holt (1962) was the first to analyze multiplicative uncertainty (uncertainty in the parameters). In particular, he shows that only when policy makers can adequately anticipate the implications of the policies they adopt, they are able to apply an active stabilization policy. Otherwise, they would contribute more to the instability of the economic system than to its stability. If the way in which the economy reacts is uncertain, that is the parameters of the behavioral model of the economy are uncertain, the performance of monetary policy could be seriously affected. In this context, the certainty-equivalence principle is not valid and, hence, the central bank should consider this type of uncertainty when making policy decisions. Brainard (1967), in his classic analysis regarding uncertainty, uses a quadratic utility function for the policy maker, similar to that of Theil (1964), to study the effect of uncertainty in shocks and parameters. He finds that if the only source of uncertainty is associated to shocks, the certainty-equivalence principle is valid. However, when the reaction of the economy to policy actions is unknown (i.e., the model feedback parameters), the behavior of the Central Bank is seriously affected and, in particular, makes it optimal to respond more cautiously to changes in the economic system.

This result has important practical implications in the conduct of monetary policy, since it indicates that it could be optimal for policy makers not to expect to completely eliminate the gap between the observed objective variable and its target value, in a particular period. This could be interpreted as a justification for a gradual monetary policy. Although Brainard's (1967) result has been widely discussed in the literature (see Blinder, 1998) and is quite intuitive, it cannot be generalized. As a matter of fact, even though there are papers (Martin and Salomon, 1999; Sack, 2000)[4] that give empirical validity to Brainard's (1967) result, there are studies that show that such result depends on the model specification. For example, Soderstrom (2002) shows that in situations where the coefficients of the lagged variables in the model are subject to uncertainty, the optimal policy for the central bank could be to react more aggressively[5].

The study of data uncertainty is relatively new in the literature on monetary policy. As a matter of fact, academics and policy makers only recently have invested resources in this area studying the properties of real-time data and its implications on policy decisions (Bernhardsen et.al., 2005). The pioneering work of Croushore and Stark (2001) set an early framework for the subject. In particular, they were the first to construct a database that provides a snapshot of the macroeconomic data available at any given date in the past with the objective of showing the implications of forecasting with revised and real-time data. In the database, the data of a particular date is known as "vintage" and "real-time data set" is the collection of the vintages. This methodology has been used in various empirical applications, which have primarily focused on developed countries. Examples of such applications, regarding the implications of real-time data for monetary policy, can be found in Orphanides and van Norden (1999) and Orphanides (2001).[6] This literature highlights that the moment at which the data are obtained, their availability, and reliability for empirical evaluation of policy rules, is crucial for monetary policy performance since they condition the decisions of the policy makers (Ghlysels, 2002). In this regard, Rudebush (2001) and Bernhardsen et.al. (2005) argue that the new information that the central banks obtain from one policy meeting to the next does not justify drastic changes in its instrument, which can lead to very slow responses to particular economic events.

One of the variables that summarize the actual state of the economy and that is, therefore, crucial for monetary policy decisions, is the output gap. Naturally, if potential output measures are not reliable, policy decisions may fail to react to the true economic conditions and may instead reflect measurement

[4] Both papers estimate a VAR model, the first for England and the second for the United States. They show that the fact of incorporating multiplicative uncertainty in the model could explain the preference for gradualism in the actual behavior of the Central Bank.

[5] Other examples in support of the argument that multiplicative uncertainty does not necessarily lead the central bank to behave more cautiously can be found in Gianoni (2002) and González and Rodríguez (2003).

[6] For an excellent literature review on the issue for the case of the United States see Kozicki (2004).

error. Along these lines, Orphanides and van Norden (1999) argue that the output gap is associated with important components of uncertainty since there are at least three types of problems typically faced by the central banks when evaluating the business cycle with real-time data. First, output data are revised continuously. Second, methods to estimate potential output provide in general different results. When trend output is used as a proxy, different filtering procedures also yield a variety of outcomes; and this problem is particularly critical with the end of sample estimates that are, precisely those relevant for policy decisions.[7] Third, a future evaluation of output data can indicate that the economy has experienced a structural change, which could have not been revealed by real-time data.

Following Zhang and Semmler (2005) and to provide an example of the concepts previously mentioned, we consider the following economic model that is standard in the literature of optimal rules of monetary policy:

$$\min_{\{u_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \rho^t L(x_t, u_t) \tag{1}$$

subject to:

$$x_{t+1} = f(x_t, u_t, \varepsilon_t) \tag{2}$$

where $\rho$ is the discount factor bounded between 0 and 1, $L(x_t, u_t)$ is a loss function of an economic agent (in this case, the central bank), $x_t$ is the vector of state variables, $u_t$ is the vector of control variables (the policy instrument), $\varepsilon_t$ is the vector of shocks and $E_0$ is the mathematical expectation operator based on the initial values of the state variables. This kind of model represents the basic framework of monetary policy analysis and control used by Clarida et.al. (1999), Svensson (1997, 1999), and Beck and Wieland (2002), where the constraints in equation (2) are the Phillips curve, the IS curve, and the interest rate parity condition, (Svensson, 2000).

Given the state equations in (2), the central bank's problem consists in deriving a path for its instrument (the control variable $u_t$) that satisfies equation (1). The question that arises, however, is whether the state equations can be correctly specified with time series estimates. Given the previous discussion, the response to this question is negative, since these equations can be subject to a high degree of uncertainty caused by shocks $\varepsilon_t$, by parameter uncertainty and by data uncertainty used in the estimations. This is particularly important since the optimal monetary policy rules are derived from the

[7] Kuttner (1994) and St-Amant and van Norden (1998), using final output data and different methods to estimate its trend, found that there were substantial differences in the estimations of these trends using final data.

solution of the previous problem[8] and, hence, these rules depend on the parameters of the state equations. Thus, if the parameters in the model are uncertain, the estimated "optimal" monetary policy rule could be unreliable.

In summary, the brief literature review presented in this section shows that in general the different types of uncertainty (additive, multiplicative, and data uncertainty) have important and different implications for the conduct of monetary policy. In particular, when the economy is faced with uncertainty in the shocks or additive uncertainty the central bank could eventually behave as if it has total certainty with respect to the results of its policy (certainty-equivalence principle). This result, however, depends on the type of assumptions adopted regarding the behavior of the central bank (its preferences) and the structure of the economy since this principle is only valid in a linear-quadratic world and depends on whether the monetary authority behaves optimally or not. Regarding multiplicative uncertainty or uncertainty in the parameters, the fact that the central bank does not know how the economy reacts to its policies would justify, in principle, a preference for more gradualism in the conduct of monetary policy. Nonetheless, there is no consensus regarding this result since the literature has shown diverse implications that depend on the assumptions that are adopted in a particular model (in some cases, the implications point to more aggressiveness in response to policy). Finally, when the data are uncertain, either because they are unknown at the moment that policy decisions are made, because they could have measurement errors (being that they are revised previously), or simply because they are unobservable, the policy decisions are seriously conditioned to the information available. Hence, sudden changes in policy when a new set of information is known could not be justified (the actual information could show an erroneous notion of the actual state of the economy). Hence, the literature has sought monetary policy rules that are "immune" to this type of uncertainty, for example, utilizing output growth rates or unemployment level rates (as opposed to the gap with respect to its natural value).

# 3. Data uncertainty: the output gap

In analyzing the quantitative relevance of data uncertainty in the case of Chile, we focus on the output gap – defined as the difference between actual (measured) GDP and its trend - for the 2000-2006 period. This period was chosen for two particular reasons: (1) the availability of historical information of the output series publications at each moment in time; and (2) this is the period in which the Central Bank of Chile adopted a *full-fledged inflation targeting* scheme to conduct its monetary policy. We use real-time data (i.e., data available to the central bank when making policy decisions) and various well-known methods to estimate the output trend. For each method we analyze the behavior of the end-of-sample

[8] See, for example, Svensson (1999).

output gap estimates, which are relevant for policy decisions, as well as the revisions of these estimates across time. We present the statistical properties of the revisions and verify the reliability of the estimates for each method. This section is divided into two subsections: the first describes the methodological issues related to the construction of the output gap with real-time data and the detrending methods; and the second one presents the results of the estimates and their implications.

## 3.1 Methodological issues

Monetary policy decisions are typically based on real-time data, which are classified as preliminary data (Bernhardsen et.al., 2005). This is also true, to a lesser degree, for long-past historical data. The preliminary nature of the data calls for it to be in constant revision and the reasons for these revisions can be, among others, of an informative nature or of a methodological nature. As suggested by the Central Bank of Chile in its Monetary Policy Report (IPoM) of September 2004, the revision of data is motivated by: the inclusion of new basic information (new sources of information or the improvement of these sources); the recalculation of the estimates (revisions attributed to new estimates);[9] methodological improvements (due to changes in statistical methods, concepts, definitions or classification); and error correction, either in the basic sources or in the calculations. One of the variables that encompasses the actual state of the economy and which is key for monetary policy decisions is the output gap. Given that at the time policy decisions are made this variable is estimated using preliminary output data, it is necessary to assess the degree of reliability of these estimates.[10] For this assessment, we use real-time data to replicate the available information for the policy makers at every point in time. Thus, we simulate the actual environment of the monetary policy setting process (Ghlysels, 2002).

To analyze the reliability and the statistical accuracy of the output gap estimates commonly used in the literature we follow the methodology proposed by Orphanides and van Norden (1999). This consists of measuring, at each point in time, the degree in which the output gap estimates vary when the data are revised using the different output gap estimation methods. This allows us to capture the effects caused by data revisions and the misspecification of statistical models used to estimate the output trend. The advantage of this methodological approach is that it does not require a priori assumptions on the true structure of the economy or on the process that generated the observed output time series. This approach also has certain limitations. Data revisions are being analyzed comparing each level of output observed at the end of the sample with the "final output", however, there could still be measurement errors.

[9] This refers to the updating of seasonal factors or of the base period used in the constant price estimates.

[10] As a matter of fact, if the output gap measures are not reliable it could be advantageous for the central bank, in some situations, to base their monetary policy decisions on information regarding output growth (Orphanides et.al., 2000; Bernhardsen et.al., 2005).

In the approach of Orphanides and van Norden (1999) there are two key definitions: the "final" and the "real-time" estimates of the output gap. The final estimate of the output gap is simply the difference between the last available vintage of output data and its trend (obtained via a detrending method). The real-time estimate of the output gap, on the other hand, is a time series consisting of the last observed estimate of the output gap constructed as the difference between the observed output *for each point in time* (each vintage) and its trend. The estimation in real-time for each period $t$ contains all the revisions available up to that period and represents the estimate that the central bank may have calculated at the time when policy decisions were made. Formally, assuming that we have access to the observed output series published at each point in time during $N$ periods we would have a matrix of the form $\left(y^1, y^2, ..., y^N\right)$, where each $y^i$ (with $i = 1,..., N$) is a column vector that contains the time series of the output and each column is an observation (row) shorter than the one that follows it.[11] If $f^{dt}(\cdot)$ is a function that detrends the time series $y$, the final estimate of the output gap is given by:

$$gap^{final} = \ln(y^N) - \ln(f^{dt}(y^N)) \tag{3}$$

On the other hand, if we define the function $\ell(\cdot)$ as one that extracts the last real observation of the column vector $y^i$ we have the real-time estimate of the output gap:

$$gap^{real-time} = \ln(\ell(y^1), \ell(y^2), ..., \ell(y^N)) - \ln(\ell(f^{dt}(y^1)), \ell(f^{dt}(y^2)), ..., \ell(f^{dt}(y^N)))' \tag{4}$$

The difference between the final output gaps and the real-time output gaps represents the *total revision* of the estimates at each point in time. The statistical properties of these series of revisions will assist in evaluating the reliability and accuracy of the output gap estimates. For the estimates drawn from equations (3) and (4) it is necessary to define the function $f^{dt}(\cdot)$ (the detrending method) given that in practice neither the true potential output of the economy nor its data generating process are known. This is important since these methods in general provide quite different results. In the case of Chile, Gallego and Johnson (2001) find that the set of methods used to estimate the trend component of output provide a wide range of estimates. Therefore, besides the revisions in the data, the method chosen also constitutes a source of uncertainty.

[11] In the matrix $\left(y^1, y^2, ..., y^N\right)$ we consider the missing observations as non-real numbers.

A detrending method decomposes real output (measured in logarithms) $y_t$ into two components: trend ( $y_t^T$ ) and cycle ( $y_t^C$ ) such that $y_t = y_t^T + y_t^C$ . We consider five alternative univariate methods that have been widely used in the literature:[12] (1) the Hodrick-Prescott filter; (2) the Baxter-King filter; (3) the Christiano-Fitzgerald filter; (4) the quadratic trend; and (5) Clark's method based on the unobservable components model.[13] Table 1 summarizes these methods and the models they are based on. We focus only on univariate techniques of detrending, since the use of multivariate techniques requires the compilation of information on the data that is not revised (in real-time) for each possible regressor in the model. Hence, the conclusions that are derived from the analysis correspond only to the evaluation of the univariate filters utilized here and cannot be applied to other alternative methods such as those used by the Central Bank of Chile[14] and in some other papers for Chile (see for example Gredig, 2007 and Fuentes et al., 2007).

***Table 1: Alternative Methods to Calculate the Output Trend***

| | | |
|---|---|---|
| HP | Hodrick-Prescott ( $\lambda = 1600$ ) | $y_t^T = \arg\min \sum_{t=1}^{T} \left\{ \left( y_t - y_t^T \right)^2 + \lambda \left( \Delta^2 y_{t+1}^T \right) \right\}$ |
| BK | Baxter-King (6,32)[15] | $y_t^T = \sum_{c=1}^{q+1} \omega^{BK}(1,c) y_{t+1-c} + \sum_{c=2}^{q+1} \omega^{BK}(1,c) y_{t+c-1}$ <br> $t = q+1,\ldots,n-q$ |
| CF | Christiano-Fitzgerald (6,32,1,0,0)[16] | $y_t^T = \sum_{c=1}^{q+1} \omega^{CF}(1,c) y_{t+1-c} + \sum_{c=2}^{q+1} \omega^{CF}(1,c) y_{t+c-1}$ <br> $t = q+1,\ldots,n-q$ |
| QT | Quadratic Trend | $y_t = \alpha + \beta t + \gamma t^2 + y_t^C$ |

[12] See Orphanides and van Norden (1999) for an extensive revision of the detrending methods and its principal advantages and disadvantages.
[13] See Gallego and Johnson (2001) for an interesting compilation of the use of these methods in different central banks of the world.
[14] The current approach used by the Central Bank of Chile to estimate the output gap is based on the production function.
[15] The series of numbers 6 and 32 represent the minimum and maximum of the desired oscillation period, respectively, for quarterly data.
[16] The series of numbers 6 and 32 have the same interpretation as in the Baxter-King filter. On the other hand, the series of numbers 1,0,0 represent the existence of unit roots, without drift and symmetric filter, respectively.

| | | |
|---|---|---|
| Clark | Unobserved Components | $y_t = y_t^T + y_t^C$<br>$y_t^T = g_{t-1} + y_{t-1}^T + \nu_t$<br>$g_t = g_{t-1} + \omega_t$<br>$y_t^C = \delta_1 y_{t-1}^C + \delta_2 y_{t-2}^C + e_t$ |

The Hodrick-Prescott filter is perhaps one of the most popular detrending methods and it is based on the choice of the trend that minimizes the variance of the cyclical component of the series; it is subject to penalization for variations in the second difference of the cyclical growth component (Hodrick and Prescott, 1997). Both the Baxter-King filter and the Christiano-Fitzgerald filter are based on the smoothing of series using weighted moving averages. The fundamental difference between both, for the case of symmetric filters as considered in this paper, lies in the choice of the objective function that defines the weights (Baxter and King, 1999; Christiano and Fitzgerald, 2003). Moreover, the Christiano-Fitzgerald filter imposes the restriction that the filter weights add up to zero when unit roots are considered. On the other hand, the quadratic trend is a method of deterministic components that assumes that the behavior of the trend series is triggered by a second order polynomial. Hence, this method is flexible at the moment of detecting slow trend changes[17]. Finally, the unobserved components model allows us to specify the data generating processes for the output time series and use these to identify the trend and cyclical components. In the particular case of the model proposed by Clark (1987), it is assumed that the trend component follows a random walk process with drift and the cyclical component follows an AR(2) process. The main advantage of this type of model is that it allows a richer short-term dynamic specification for the model.

## 3.2 Results

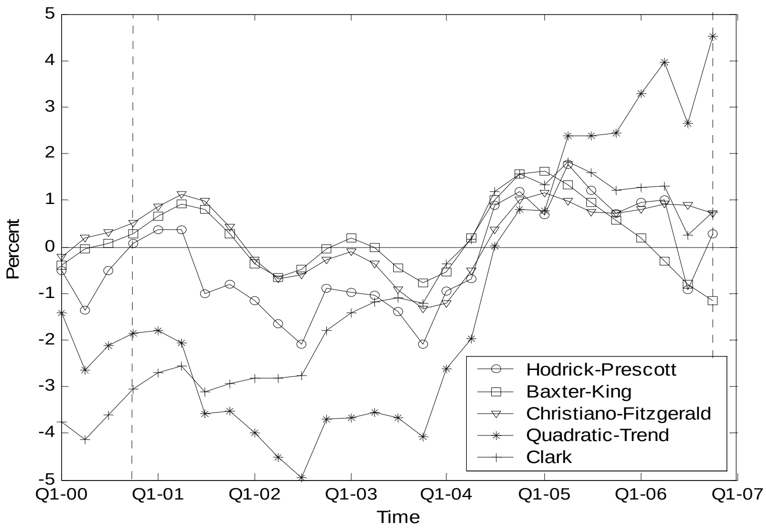
The output data observed at each point in time were constructed using data compiled from the monthly publications (bulletins) of the Central Bank of Chile. For each new statistical entry in which a new output record was published an output series was constructed, which included the revisions of the data published before.[18] For the quantitative evaluation of uncertainty in the output gap estimates, we

[17] Its simplicity has made it quite valuable for empirical applications related to monetary policy (for example, Clarida et.al, 1998). However, its use has generated much controversy due to the argument that better modeling of the output requires statistical components in the model.
[18] In some cases the revisions were observed for one or two quarters back and in others, such as the periods in which there are base changes, the revisions were performed on the complete series. The Central Bank revised the national accounts and changed the base year in two separate occasions during our sample period. The first time was in the

consider the period between the first quarter of 2000 and the last quarter of 2006. Nonetheless, the output gap estimates were calculated based on information since 1986.[19] Hence, the first time series we use covers the period between the first quarter of 1986 and the first quarter of 2000. The series that follows contains an additional quarter not included in the previous series and this occurs successively up until the last series, which is comprised of the complete period, that is, from the first quarter of 1986 to the last quarter of 2006. Each output series was seasonally adjusted using the X-12 ARIMA procedure employed by the Central Bank of Chile. Hence, the series reflect both the revisions and the re-estimation of seasonal factors. Finally, the series published in the last quarter of 2006 is our "final" series of output, although we are aware that this series contains data that will be revised in the future.

The compilation of the information described above produced a total of 28 output series for each point in time. We apply the five detrending methods to each of these estimates to calculate the output gap. Following the methodology applied by Orphanides and van Norden (1999), our final estimates are the output gap for the last available series and our real-time estimates are the series constructed with the last observation of each of the output gaps estimated with the 28 series. Figures 1 and 2 illustrate these estimates using final and real-time data.

***Figure 1: Output Gap Estimates for the Chilean Economy with Final Data***

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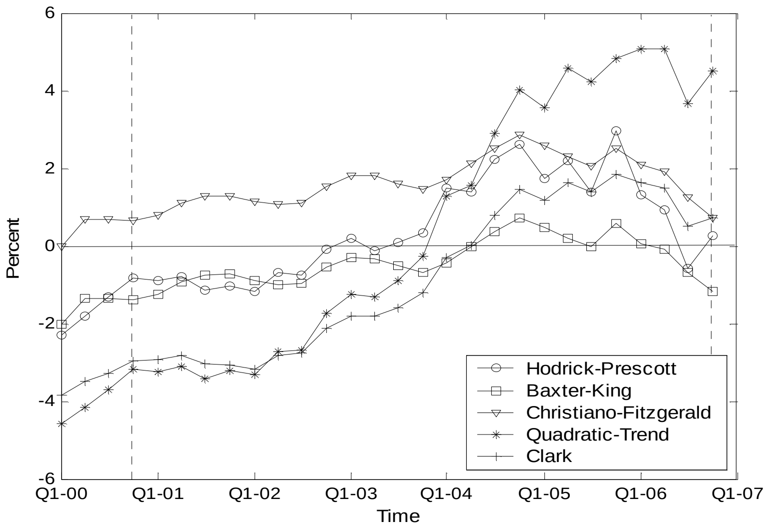
fourth quarter of 2001, in which the base year changed from prices of 1986 to prices of 1996, and the second one was done in the last quarter of 2006 changing the base year to 2003 (the vertical dotted lines in Figures 1 to 3 show these changes).

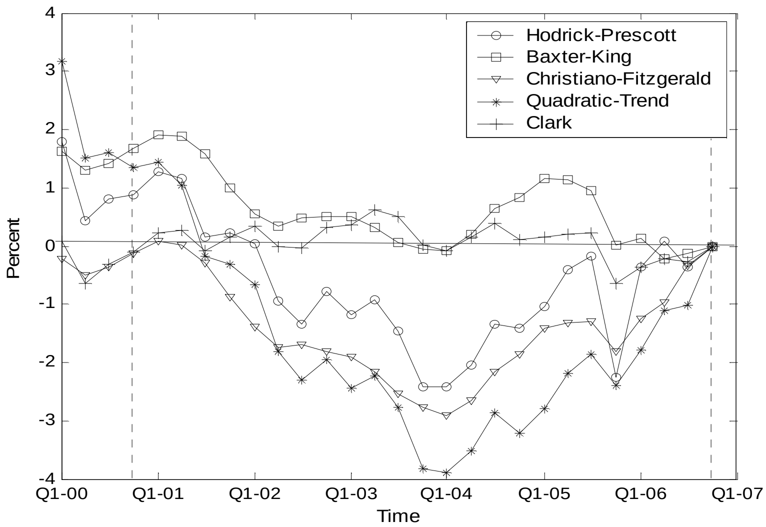
[19] For a statistical filter to produce reasonable results we need at least a complete cycle in the series, which implies that long time series are necessary.

*Figure 2: Output Gap Estimates for the Chilean Economy with Real-Time Data*




From figures 1 and 2 we observe that most of the estimations generated by the different detrending methods behave similarly in terms of their trajectories. This is true for both the estimations using final data and those using real-time data. The only exception is the estimation of the output gap based on the quadratic trend. However, despite the comovements observed in the different series, the magnitude of the changes varies considerably from one method to the other. In the same way, the different methods produce a wide range of output gap estimates. The average difference between the highest and lowest estimates is 6% with final data and 12% with real-time data. The order of magnitude of these differences is considerable since they are quite superior to the difference between the highest and the lowest points of the business cycle within the period considered (approximately 5% for both types of data and for a majority of filters). The average dispersion that exists between methods is also important and reaches 2.3% when using final data and 4.3% in the case of real-time data. Another important feature of the estimations using final data is that these tend to be clustered between the fourth quarter of 2004 and the third quarter of 2005. In addition, these estimates remain relatively close towards the end of the period of analysis with the exception of the output gap based on the quadratic trend. This latter feature is not observed with real-time estimates. To have a qualitative idea of the importance of data revision, figure 3 shows the difference between the estimates with final data and those with real-time data for the five detrending methods. This difference represents the total revision in the output gap.

*Figure 3: Total Revisions in the Output Gap for the Chilean Economy*




As shown by figure 3, the magnitude of the revisions is also important and differs substantially between the filters used (the average dispersion of revisions between different measures is 2.8%). The most extreme cases are observed in early 2004, where revisions of the HP, CF and quadratic trend methods were the most important in the entire sample. This is due to the fact that these filters do not adequately capture the turning point of the output gap in that period (see figures 1 and 2) and, therefore, suggests that real-time estimates were imprecise. This is not the case for the BK and Clark methods, which in the same point in time observe practically null revisions. On the contrary, the most important revisions for these last two filters were observed at the beginning of the sample. To better understand the differences between the estimates with final data and those with real-time data, we present descriptive statistics of the output gap estimates and of the revisions for the five filters in tables 2 and 3, respectively. Figure 4 shows the time behavior of all these estimates.

*Table 2: Descriptive Statistics of the Output Gap Measures calculated with Final and Real-Time Data*

| | **Mean** | **Abs. Value** | **Std** | **Min** | **Max** | **Corr** |
|---|---|---|---|---|---|---|
| | | | Hodrick-Prescott | | | |
| Final Estimates | -0.003 | 0.010 | 0.011 | -0.021 | 0.018 | 1.000 |
| Real-Time Estimates | 0.002 | 0.012 | 0.014 | -0.023 | 0.030 | 0.611 |
| | | | Baxter-King | | | |
| Final Estimates | 0.002 | 0.006 | 0.007 | -0.012 | 0.016 | 1.000 |
| Real-Time Estimates | -0.005 | 0.007 | 0.007 | -0.020 | 0.007 | 0.561 |
| | | | Christiano-Fitzgerald | | | |
| Final Estimates | 0.002 | 0.007 | 0.008 | -0.013 | 0.012 | 1.000 |
| Real-Time Estimates | 0.015 | 0.015 | 0.007 | 0.000 | 0.029 | 0.203 |
| | | | Quadratic-Trend | | | |
| Final Estimates | -0.012 | 0.028 | 0.029 | -0.050 | 0.045 | 1.000 |
| Real-Time Estimates | 0.001 | 0.031 | 0.035 | -0.046 | 0.051 | 0.841 |
| | | | Clark | | | |
| Final Estimates | -0.010 | 0.019 | 0.020 | -0.041 | 0.018 | 1.000 |
| Real-Time Estimates | -0.011 | 0.020 | 0.020 | -0.039 | 0.019 | 0.988 |

*Table 3: Descriptive Statistics of the Total Revisions in the Output Gap*

| | **Mean** | **Abs. Value** | **Std** | **Min** | **Max** | **AR** |
|---|---|---|---|---|---|---|
| Hodrick-Prescott | -0.005 | 0.010 | 0.011 | -0.024 | 0.018 | 0.700 |
| Baxter-King | 0.007 | 0.007 | 0.007 | -0.002 | 0.019 | 0.875 |
| Christiano-Fitzgerald | -0.013 | 0.013 | 0.009 | -0.029 | 0.001 | 0.939 |
| Quadratic-Trend | -0.013 | 0.020 | 0.019 | -0.039 | 0.032 | 0.842 |
| Clark | 0.000 | 0.002 | 0.003 | -0.006 | 0.006 | 0.473 |

Comparing the results in tables 2 and 3 we observe that, on average, total revisions are similar to or greater than the output gap estimates for all filters used.[20] Something alike occurs with the average gap in absolute value. This confirms the previous discussion since the revisions are always significant in magnitude regardless whether the economy is in a recession or expanding. With respect to the minimum and maximum points of the cycle, the estimations with final and real time data coincide with the minimum values of the gap only in the case of Clarks method (see figure 4; panel e), while with the BK filters, quadratic trend, and Clark's method the estimations coincide with the maximum values (see figure 4; panels b, d, e). This suggests that most of the methods fail to identify the magnitude of the recessive periods. The last column of table 2 shows the correlation coefficients between final data estimates and

[20] This result is qualitatively similar to that found in Orphanides and van Norden (1999) for the US economy.

real-time data estimates for each filter. The highest correlations are observed for the Clark and the quadratic trend methods (over 0.8), while the CF and BK filters produce the lowest correlations. Another important element is the degree of persistence of the revisions since as the revisions persist over time, the discrepancies between the final and real-time estimates tend to remain or disappear slowly in time. The last column of table 3 reports the estimated first order autocorrelation coefficients for total revisions which indicate, with the exception of the Clark filter, that these revisions are highly persistent.

The question yet to be responded is whether the measures of the output gap constructed with real-time data are reliable.[21] Since the different methods vary substantially with respect to the size of the cyclical component, it is more convenient to compare the reliability of the real-time estimates using independent scale measures. Table 4 presents the reliability measures used by Orphanides and van Norden (1999).

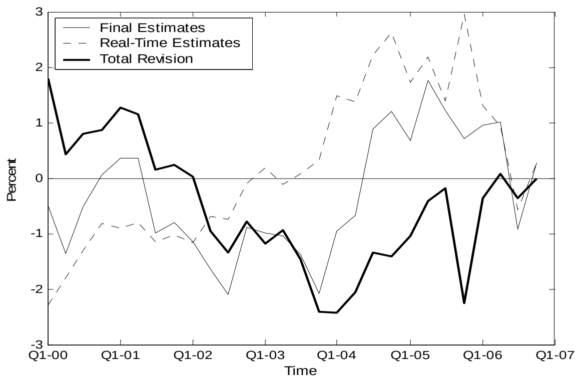
***Table 4: Descriptive Statistics of the Reliability Measures for Alternative Different Filters***

| | **Corr** | **N/S** | **Opsign** | **Xsize** |
|---|---|---|---|---|
| Hodrick-Prescott | 0.611 | 1.055 | 0.286 | 0.500 |
| Baxter-King | 0.560 | 0.902 | 0.321 | 0.536 |
| Christiano-Fitzgerald | 0.203 | 1.229 | 0.393 | 0.750 |
| Quadratic-Trend | 0.841 | 0.650 | 0.071 | 0.214 |
| Clark | 0.988 | 0.156 | 0.000 | 0.036 |

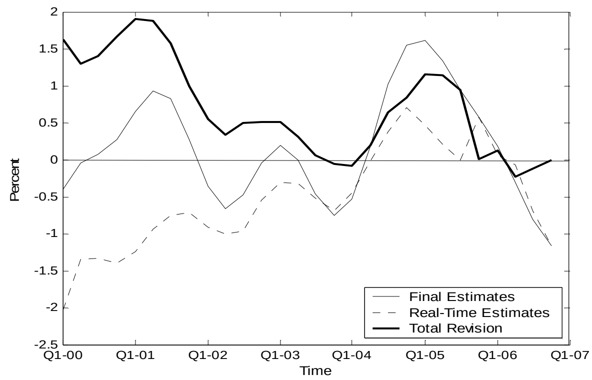
[21] We seek reliability measures as it relates to quantifying the difference between the final estimates and the real-time estimates. Hence, it does not indicate anything regarding the reliability of each method as tools for the estimation of the "true" output gap (Bernhardsen et.al., 2005).

***Figure 4: Estimation of the Output Gap and Total Revisions using Final and Real-Time Data for the Five Alternative Filters***

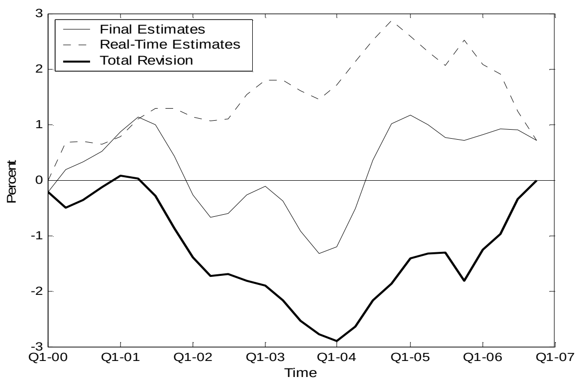
*(a) Hodrick-Prescott*



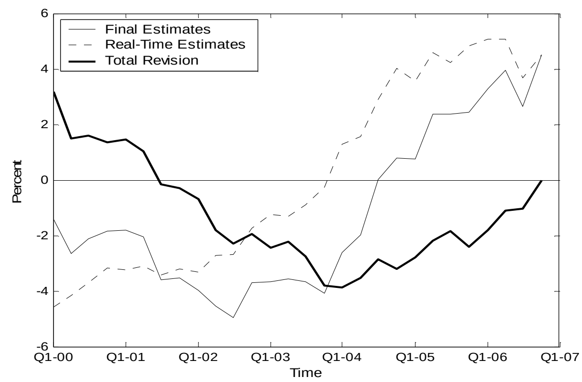

*(b) Baxter-King*



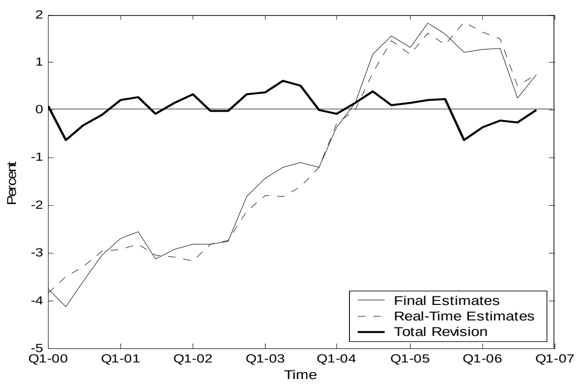

*(c) Christiano-Fitzgerald*




*(d) Quadratic-Trend*




*(e) Clark*



In the first column of table 4 we present the correlation between final and real-time series for each detrending method. The other three indicators in table 4 measure in different ways the relative importance of the revisions (the ideal value for these three indicators is zero). The N/S indicator is the ratio of the standard deviation of the revision to that of the final estimate of the output gap and approximates the noise-to-signal ratio. The OPSING indicator shows the frequency with which the real-time estimates of the output gap reveals a different sign when compared to the final estimates. Finally, the XSIZE indicator shows the frequency with which the absolute value of the revision exceeds the absolute value of the final estimates of the output gap. The Clark and the quadratic trend methods reveal smaller noise levels, smaller frequencies in observations with errors in the sign and with significant size in the revision. On the other hand, the CF filter shows the poorest performance under these reliability measures.

Summing up, results above show that, in general, revisions of the output gap seem to be important and persistent for the period considered, and that the correlations between the final and real-time estimates of the output gap are relatively low. Nonetheless, the Clark method brings the most favorable statistics. The analysis also reveals that this later method is the most reliable with real-time data.[22] Comparing our results with those of Orphanides and van Norden (1999) for the US economy, we find that in general the different reliability measures produce similar values. In general, these results imply that caution and judgment should be taken when assessing the level of the real-time estimates of the output gap, at least with the methodologies utilized here. Additionally, our results should be considered a lower bound to measurement errors that could be present in the output gap estimates because comparisons are made with respect to a measure of the final output gap that could contain unrevised data.

# 4. Additive and multiplicative uncertainty

To focus on the empirical importance of additive and multiplicative uncertainty we resort to data for the 1990 to 2006 period but place some emphasis on the sub-sample 1999-2006, the *full-fledged inflation targeting* period. We adopt a slightly modified version of the *forward-looking* specification of Svensson (2000) and Al-Eyd and Karasulu (2006) to estimate the behavioral equations of a small open economy, as is the case of Chile (i.e, the aggregate demand, the Phillips curve and the real uncovered interest parity condition). Like in Zhang and Semmler (2005), we do not include a monetary policy rule in this specification given that the objective of the paper is to analyze the primary sources of uncertainty faced by

[22] Additionally, as a robustness test, we calculated the reliability measures in real time utilizing output gap estimations with non-seasonally adjusted data and seasonally adjusted data through the seasonal dummy variables. Conclusions do not change (for details on the results see appendix A). This exercise was done to verify whether the re-estimation of the seasonal factors, which is not present in the non-seasonally adjusted data and is constant when we use seasonal dummy variables, substantially influences on results.

the Central Bank, which is associated to the structure and behavior of the economy.[23] To capture the sources of uncertainty, we estimate the model with time-varying parameters, assuming that shocks have state-dependent variances (two states) and that their behavior follows a first order Markov process. This strategy allows us to decompose the conditional variance of the forecast error into two components: one associated with the parameters (multiplicative uncertainty) and the other attributed to the shocks in the model (additive uncertainty).

## 4.1 Methodological issues

The existing literature that models additive and multiplicative uncertainty typically use models that explicitly consider stochastic volatility potentially present in the errors (heteroscedasticity) and time-varying parameters (Zhang and Semmler, 2005). Among papers that have explicitly dealt with parameter uncertainty we find Cogley and Sargent (2001), who studied the inflation dynamics of the United States in the post World War II period using a Bayesian VAR with time-varying parameters; and Semmler et.al. (2005), who estimated the Phillips curve and a monetary policy Taylor rule for the Euro Zone also with time-varying parameters. Both authors found that there are substantial changes in the model parameters. However, even though the evidence encountered when using models with time-varying parameters suggests the existence of important degrees of uncertainty, in the modeling process this analysis cannot be separated from additive uncertainty. When additive uncertainty is not considered, volatility in the parameters could be exaggerated when it is indeed captured (Sims, 2001). An example can be found in Sims and Zha (2006), who study regime changes in the US economy dynamics and find, contrary to Cogley and Sargent (2001), evidence in favor of stable model dynamics but unstable variance of the disturbances. Thus, Cogley and Sargent (2005) modify their original model considering time-varying parameters and stochastic volatility and also find the existence of regime changes. More recent examples of the estimation of Taylor rules with time-varying parameters and stochastic volatility can be found in Kim and Nelson (2006) and Zampolli (2006).

To incorporate both types of uncertainty, additive and multiplicative, we follow the Zhang and Semmler (2005) approach. We use a model with time-varying parameters and shocks that have state-dependent variance. Contrary to Cogley and Sargent (2005), who assume that the variance of the shocks changes each period, we assume that the variance has only two states (high and low) and follows a Markov process, as in Sims and Zha (2006).[24] This specification, besides having the advantage of dealing

[23] Moreover, the optimal monetary policy feedback parameters will depend on the structure and behavior of the economy.
[24] These authors assume that the variance of the regression errors follow a Markov process with three states.

with both types of uncertainty in the same model, allows the decomposition of the variance of the forecast error into two components: one associated to additive uncertainty and the other linked to multiplicative uncertainty (Kim, 1993).

The specification we use for the behavioral equations of the economy is a slightly modified version of the specification of Svensson (2000) and Al-Eyd and Karasulu (2006); it is a neo-Keynesian version for a small open economy comprised of the IS curve (aggregate demand), the short-run supply curve (Phillip's curve), and the real uncovered interest parity condition (UIP). As opposed to these authors, we allow deviations of the UIP because of imperfections in the capital markets, capital controls, speculative bubbles, etc. As it is usual in the modern Dynamic Stochastic General Equilibrium (DSGE) literature, the deviations in the UIP are modeled introducing a backward-looking component in the original specification of Svensson (2000) and Al-Eyd and Karasulu (2006). Thus, the behavioral equations of the economy can be written as:

$$y_t = \theta_{1t} y_{t-1} + \theta_{2t} E_t[y_{t+1}] + \theta_{3t} r_{t-1} + \theta_{4t} q_{t-1} + \varepsilon_t^d \tag{5}$$

$$\pi_t = \varphi_{1t} \pi_{t-1} + \varphi_{2t} E_t[\pi_{t+1}] + \varphi_{3t} y_{t-1} + \varphi_{4t} q_t + \varepsilon_t^s \tag{6}$$

$$q_t = \gamma_{1t} E_t[q_{t+1}] + \gamma_{2t}(r_t - r_t^f) + \gamma_{3t} q_{t-1} + \upsilon_t \tag{7}$$

where $y_t$ represents the real output gap, $\pi_t$ is the inflation rate, $r_t$ is the short-term real interest rate, $q_t$ is the real exchange rate and $r_r^f$ is the foreign real interest rate, observed in period $t$. On the other hand, $E_t[y_{t+1}]$, $E_t[\pi_{t+1}]$ and $E_t[q_{t+1}]$ represent the expectations for period $t+1$ of the output gap, the inflation rate, and the real exchange rate, respectively, conditional on the information available at period $t$ ($E_t$ is the expectations operator). The terms $\varepsilon_t^d$, $\varepsilon_t^s$ and $\upsilon_t$ are whose variances are state-dependent. The first two are aggregate demand and supply shocks, respectively, and the third one is associated with the exchange market. In words of Al-Eyd and Karasulu (2006), this last disturbance term could be interpreted as a risk premium that captures the effects of the unobservables, such as the exchange market sentiments. Finally, $\theta_{it}$ (with $i = 1,2,3,4$), $\varphi_{it}$ (with $i = 1,2,3,4$) and $\gamma_{it}$ (with $i = 1,2,3$) are the time-varying parameters.

Two interesting observations can be made to this specification. First, the explicit inclusion of the exchange rate in the modeling process is relevant for an economy such as Chile whose Central Bank uses inflation targeting as a monetary policy framework. Compared to the closed economy models, an important additional transmission channel of monetary policy is introduced and the external shock effect on the domestic economy is incorporated. Second, the specification incorporates both *forward-looking* and *backward-looking* terms (hybrid model), for which there is empirical backing at least in the case of the Phillips curve (Caputo et.al., 2006, and Céspedes et.al., 2005). F*orward-looking* terms can be justified by appealing to sticky price models of the Calvo (1983) type, whose wage (price) setting mechanism is built-in in a fraction of Chilean labor contracts.

However, the inclusion of *forward-looking* components brings the problem of how they are measured or approximated, a choice that can have important implications for estimation properties (consistency). The literature has proposed various ways to deal with these variables and the most appropriate estimation techniques used in each case. An obvious option is to use ex-post data, that is, approximate the expectation variables with their respective observed future values. Even though this option is operationally simple, it generates an endogeneity bias in the estimation of the model parameters, which leads to inconsistent estimates (Kim and Nelson, 2006).[25] [26]

Galí and Gertler (1999), Roberts (2001) and Galí et.al (2005) proposed a methodology to deal with the endogeneity problem. It is based on using ex-post data for the *forward-looking* component of the model and estimating by the Generalized Moments Method (GMM) to instrumentalize the expectations. The use of the GMM techniques to estimate the Phillips curve, as well as the *forward-looking* Taylor rules is very common in the literature[27]. Along these lines, Kim (2004, 2006) recently proposed the application of instrumental variables for the estimation with endogenous regressors using time-varying parameter models and regime changes. More specifically, this methodological proposal solves the endogeneity problem applying the Kalman filter in a two-stage Heckman (1976) estimation.[28] The specification of the behavioral equations in (5) to (7) under Kim's (2004, 2006) methodology can be rewritten in a state-space form as follows:

[25] This is relevant given that one of the objectives of the article is to study precisely parameter uncertainty.

[26] Another straight-forward option (Roberts, 1995), is to use data from expectation surveys to construct a proxy variable of expectations. This alternative, however, has two potential problems: the first one is associated to the availability of long period time series for the estimation; and the second one is that in general surveys are measured with error.

[27] For the case of Chile, there are various examples of papers applying this methodology, among which we find: Céspedes et.al. (2005), who estimated a hybrid Phillips curve, and Corbo (2002), who estimated a reaction function for the Central Bank.

[28] A recent application of this methodology used to estimate a forward-looking Taylor rule with ex-post data for the United States can be found in Kim and Nelson (2006).

$$
\begin{aligned}
& x_t = w_t'\beta_{1t} + v_t'\beta_{2t} + \varepsilon_t && \varepsilon_t \sim N(0,\sigma^2_{\varepsilon,S_t}) \\
& \beta_t = \beta_{t-1} + \eta_t && \eta_t \sim N(0,Q_\eta) \\
& v_t = Z_t'\delta_t + \xi_t && \xi_t \sim N(0,Q_\xi) \\
& \delta_t = \delta_{t-1} + \kappa_t && \kappa_t \sim N(0,Q_\kappa) \\
& \sigma^2_{\varepsilon,S_t} = \sigma^2_{\varepsilon,0} + (\sigma^2_{\varepsilon,1} - \sigma^2_{\varepsilon,0})S_t && \sigma^2_{\varepsilon,1} > \sigma^2_{\varepsilon,0}
\end{aligned}
\tag{8}
$$

where $x_t$ represents a vector of state variables ($y_t, \pi_t, q_t$ for the aggregate demand, the Phillip's Curve, and the UIP, respectively), $w_t$ is the vector of explanatory variables which are assumed to be exogenous or predetermined ($y_{t-1}, r_{t-1}, q_{t-1}$ for the aggregate demand, $\pi_{t-1}, y_{t-1}, q_t$ for the Phillip's Curve and $r_t - r_t^f, q_{t-1}$ in the case of the UIP), $v_t$ is a vector of endogenous explanatory variables, which are correlated with the errors of the model $\varepsilon_t$ ($y_{t+1}, \pi_{t+1}, q_{t+1}$, respectively), $Z_t$ is a vector of instrumental variables, $\beta_t = (\beta_{1t}, \beta_{2t})'$ and $\delta_t$ is a vector of time-varying parameters and $\eta_t$, $\xi_t$ and $\kappa_t$ are Gaussian errors with a matrix of variances $Q_i$ with $i = \eta, \xi, \kappa$, and $S_t$ is an unobservable indicator variable witch is equal to 1 in the high volatility state and 0 otherwise. We assume that the variance of errors $\varepsilon_t$ present two states with transition probabilities that follow a Markov process and which are expressed as: $\Pr[S_t = 1 \mid S_{t-1} = 1] = p$ and $\Pr[S_t = 0 \mid S_{t-1} = 0] = q$.

Kim (2006) proposes specifying the endogeneity in the model assuming that the correlation between the error term $\varepsilon_t$ and the standardized forecast error associated with the endogenous variables $\xi_t^*$ (i.e., the prediction error associated with the rational expectations of the agents) is constant and equal to $\rho$. On the other hand, and considering that the variance of the errors is state-dependent, Kim (2004) suggests that such correlation will also be state-dependent. Thus, the error of the model can be rewritten as $\varepsilon_t = \xi_t^{*}{}'\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t$ with $\omega_t \sim N(0,1)$. Using this last expression we can write the first equation of model (8) as:

$$
x_t = w_t'\beta_{1t} + v_t'\beta_{2t} + \xi_t^{*}{}'\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t \qquad \omega_t \sim N(0,1) \tag{9}
$$

where $\rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t$ and $S_t$ is the same indicator variable defined above. In this last equation the error of the model is independent of $v_t$ and of $\xi_t^*$. Hence, the estimation generates parameters that are consistent. For the estimation, Kim (2004, 2006) proposes the following two-stage procedure. The first stage consists in estimating the model that instrumentalizes the endogenous variables using the maximum log-likelihood method based on the forecast of the error and the conventional Kalman filter, that is:

$$\begin{aligned} v_t &= Z_t'\delta_t + \xi_t & \xi_t &\sim N(0, Q_\xi) \\ \delta_t &= \delta_{t-1} + \kappa_t & \kappa_t &\sim N(0, Q_\kappa) \end{aligned} \tag{10}$$

Then the standardized forecast error of $v_t$ is calculated as $\xi_t^* = Q_{\xi,t|t-1}^{-1/2}(v_t - Z_t'\delta_{t|t-1})$ for all $t = 1,2,...,T$. The second stage consists in using the forecast error calculated previously to estimate the following model using maximum log-likelihood techniques that combine the use of the Kalman filter and the EM algorithm proposed by Hamilton (1989, 1990)[29]:

$$\begin{aligned} & x_t = w_t'\beta_{1t} + v_t'\beta_{2t} + \xi_t^{*}{}'\rho_{S_t}\sigma_{\varepsilon,S_t} + \sqrt{1-\rho_{S_t}'\rho_{S_t}}\,\sigma_{\varepsilon,S_t}\omega_t & & \omega_t \sim N(0,1) \\ & \beta_t = \beta_{t-1} + \eta_t & & \eta_t \sim N(0, Q_\eta) \\ & \sigma_{\varepsilon,S_t}^2 = \sigma_{\varepsilon,0}^2 + (\sigma_{\varepsilon,1}^2 - \sigma_{\varepsilon,0}^2)S_t \qquad \sigma_{\varepsilon,1}^2 > \sigma_{\varepsilon,0}^2 & & \\ & \rho_{S_t} = \rho_0 + (\rho_1 - \rho_0)S_t & & \end{aligned} \tag{11}$$

Finally, the same author, in a previous paper (Kim, 1993)[30] suggests a procedure, using the estimation of the specification (8), to decompose the conditional variance of the forecast error ($f$) into two components: $f^1$, the conditional variance due to changes (lack of knowledge) in the model parameters, or multiplicative uncertainty, and $f^2$, the conditional variance given the heteroscedasticity in the error term, or additive uncertainty. In this procedure, the Kim exploits the informational structure of the model related

[29] The estimation algorithm is presented in appendixes A to D. A potential limitation of the methodology of Kim (2004, 2006) in the estimation of the behavioral equations of the economy is that he assumes that the shocks associated to each equation are independent from each other, and therefore, does not take advantage of the information contained in the correlations that could exist between each other (that is, common states). In other words, the methodology permits the estimation of each equation separately and, therefore, the different states of the shocks will not necessarily coincide for the three equations. As a matter of fact, Zhang and Semmler (2005) find very different occurrence probabilities for each state of the shocks depending on whether they are dealing with the aggregate demand or the Phillips curve, indicating that the states in the model do not coincide in the same time period.

[30] In his paper, Kim (1993) identifies the sources of uncertainty and its importance associated to the process of monetary creation in the United States.

to the probability distributions in the different states. The conditional variance due to the multiplicative uncertainty depends on the state in a previous period, while the conditional variance due to additive uncertainty depends on the state in the current period. This decomposition is quite useful since it gives us the percentage of the total variance of the forecast error that is caused by each source of uncertainty. Formally, we have:[31]

$$f_t = f_t^1 + f_t^2 \rightarrow \begin{cases} f_t^1 = (w_{t-1}, v_{t-1}) \left\{ \sum_{i=0}^{1} \Pr[S_t = i \mid \psi_{t-1}] \left[ P_{t|t-1}^i + \left(\tilde{\beta}_{t|t-1} - \beta_{t|t-1}^i\right)\left(\tilde{\beta}_{t|t-1} - \beta_{t|t-1}^i\right)' \right] \right\} (w_{t-1}, v_{t-1})' \\ f_t^2 = \sigma_{\varepsilon,S_t}^2 = \sigma_{\varepsilon,0}^2 + (\sigma_{\varepsilon,1}^2 - \sigma_{\varepsilon,0}^2)\Pr[S_t = 1 \mid \psi_{t-1}] \end{cases} \quad (12)$$

where $\tilde{\beta}_{t|t-1} = \sum_{i=0}^{1} \Pr[S_t = i \mid \psi_{t-1}] \beta_{t|t-1}^i$ and $P_{t|t-1}^i$ is the variance-covariance matrix of $\beta_{t|t-1}^i$ at state $i$.

## 4.2. Results

To estimate (8) we use quarterly data for the period defined from the first quarter of 1990 to the last quarter of 2006. The output gap, $y_t$, is the difference between the observed GDP and its trend, calculated using the HP filter. We use the HP filter since it is one of the most utilized in the literature and allows us to compare our results with those of other papers that estimate behavioral equations for Chile. Even though the Clark filter behaves best with real-time data, according to the results in the previous section, this does not imply that it is the best filter to estimate the "true" output trend. Additionally, given that the output series ends in 2006, our measure of the output gap is that which we consider as "final". Thus, the uncertainty associated with data revisions is not included in the types of uncertainty analyzed in this section[32]. On the other hand, the quarterly inflation rate $\pi_t$ is measured as the quarterly variation of the underlying consumer price index (CPIX).[33] As in Céspedes et.al. (2005), we use the CPI variation instead of the implicit deflator variation of the GDP since the latter, for the case of Chile, is measured with considerable noise and is strongly influenced by the variations in the terms of trade. Also, the Central Bank's inflation target is expressed in terms of CPI variations. In the case of the real exchange rate, $q_t$, we chose the bilateral exchange rate index with the United States. Finally, the foreign and domestic short-

[31] For details on the formal derivation of the decomposition of the conditional variance of the forecast error see Kim and Nelson (1999).
[32] The way in detrending is made may have effects on the estimations. Thus, we run a robustness analysis below.
[33] We chose the underlying index to avoid the influence of the regulated prices and of those that show significant variations.

term interest rates, $r_t$ and $r_t^f$, are defined as the monetary policy rates of Chile and the United States, respectively. All the previous data were obtained from the Central Bank of Chile database. Table 5 shows the parameters estimated using Heckman's two-stage procedure detailed in Kim (2004, 2006).[34] The parameters presented in this table are not structural parameters of the model.

***Table 5: Estimation of the Behavioral Equations***

| Aggregate Demand | | | Phillips Curve | | | Real Uncovered Interest Parity | | |
|---|---|---|---|---|---|---|---|---|
| Param. | Estimated | S.D. | Param. | Estimated | S.D. | Param. | Estimated | S.D. |
| $p$ | 0.6571 | 0.5267 | $p$ | 0.6639 | 1.5101 | $p$ | 0.9479 | 3.2325 |
| $q$ | 0.6586 | 0.0644 | $q$ | 0.8475 | 0.0501 | $q$ | 1.0000 | 0.0001 |
| $\sigma_{\eta_1^\theta}$ | 0.0697 | 0.2565 | $\sigma_{\eta_1^\phi}$ | 2.4407 | 1.0338 | $\sigma_{\eta_1^\gamma}$ | 0.0000 | 0.0026 |
| $\sigma_{\eta_2^\theta}$ | 0.0797 | 0.2441 | $\sigma_{\eta_2^\phi}$ | 1.2700 | 0.8449 | $\sigma_{\eta_2^\gamma}$ | 0.0007 | 2.3801 |
| $\sigma_{\eta_3^\theta}$ | 0.2942 | 0.2540 | $\sigma_{\eta_3^\phi}$ | 0.0000 | 0.0001 | $\sigma_{\eta_3^\gamma}$ | 0.0000 | 0.0029 |
| $\sigma_{\eta_4^\theta}$ | 0.0002 | 0.0002 | $\sigma_{\eta_4^\phi}$ | 1.6518 | 0.9554 | | | |
| $\sigma_{\varepsilon,0}$ | 0.0570 | 0.0098 | $\sigma_{\varepsilon,0}$ | 0.0329 | 0.0084 | $\sigma_{\upsilon,0}$ | 2.4467 | 0.2295 |
| $\sigma_{\varepsilon,1}$ | 0.4806 | 0.2347 | $\sigma_{\varepsilon,1}$ | 0.5497 | 1.2718 | $\sigma_{\upsilon,1}$ | 3.7539 | 0.1850 |
| $\rho_0$ | 0.5123 | 0.1594 | $\rho_0$ | 0.0010 | 0.2473 | $\rho_0$ | 0.4924 | 0.2057 |
| $\rho_1$ | 0.6324 | 0.1892 | $\rho_1$ | 0.4705 | 0.1446 | $\rho_1$ | 1.0000 | 0.2750 |
| Loglike | -64.026 | | Loglike | -80.389 | | Loglike | -109.64 | |

There are two interesting results that we would like to highlight. The first one is that variances of shocks confirm that there are two states in the three behavioral equations: a high volatility state and a low volatility state. For the aggregate demand estimations, the variance of shocks in the high volatility state is substantially greater than in the low volatility state (0.48 vs. 0.05). The difference between these variances for the Phillips curve is just as large (0.54 and 0.03 in the high and low volatility states, respectively). Similar results are obtained in the case of the UIP (3.75 vs. 2.45), even though the magnitude of the difference is not as large as in the previous two cases. Additionally, all the variances, except that

[34] In the application of the Kalman filter for the evaluation of the likelihood function we eliminated 12 observations at the beginning of the sample due to the presence of non-stationary time series in the model; see Kim and Nelson (1999).

associated with the high volatility state of the Phillips curve, are statistically significant. However, even though the difference between the variances of shocks for the UIP is not significant, the size of the variances is considerable compared to those found for the aggregate demand and the Phillips curve. The second element refers to the existing correlation between the shocks of the behavioral equations and the errors in the expectations of economic agents that also vary substantially with the states. In particular, the results suggest that in high volatility states of the shocks, agents tend to commit crucial errors in their forecasts. This fact is particularly true for the Phillips curve, where such correlation varies between 0.001 and 0.47 for both states, and for the real uncovered interest parity condition (0.49 vs. 1). In the case of aggregate demand there is also an important correlation in the high volatility state. Nonetheless, the difference between the correlations of both states is less evident than in the previous two cases. Also, the correlation coefficients are highly significant for all cases except for the one associated to the low volatility state of the shocks in the Phillips curve.

Figures 5 to 7 show the behavior over time of the structural parameters of the equations estimated in table 5. There are two series in each figure, which correspond to the relevant values of the parameters in each possible state of shocks in the model (i.e., high volatility and low volatility). In the case of the aggregate demand parameters (figure 5), there are two clearly defined periods. The first period, which ends in 1999, is marked by high instability and substantial differences between the parameters of the two states associated with the demand shocks. During this period, the average probability that the economy was in a high volatility state was 0.82 and the macroeconomic context was characterized by a substantial range of variation in the annual GDP growth rate (from 15% to below 6%) and by high inflation rates. The second period (from 1999 onward) saw a significant reduction in instability, as well as in the differences of the parameters with respect to the state of the shocks, with the exception of the parameter associated with the output gap's degree of persistence. The average probability that the economy was in a high-volatility state was only 0.10. These results suggest that the multiplicative uncertainty associated with the aggregate demand tends to decline over time. Also, the degree of persistence of the output gap ($\theta_{1,t}$) and the response of this to changes in relative prices ($\theta_{4,t}$) have declined over time, while the contrary has occurred with the degree of response to expectations ($\theta_{2,t}$) and the monetary policy interest rate ($\theta_{3,t}$). This is consistent with the logic of the inflation targeting framework.[35]

Parameters of the Phillips curve (figure 6) show a significant dependence on the state of the supply shocks. In particular, during the periods of high volatility, parameters tend to show high instability and in periods of low volatility they are much more stable. As opposed to the results of the aggregate demand

[35] This behavior intensified as of 1999 with the establishment of the full-fledged inflation targeting framework.

parameters, this dependence was maintained throughout the entire period. Hence, the state of shocks is key in explaining greater or lower degrees of uncertainty in the Phillips curve parameters. During most of the 90s, a high volatility state of shocks prevailed (with an average probability of 0.9) and therefore the relevant parameters in that period were those of the high volatility state, while in the most recent period (1999 onward) the average probability was only 0.06. Figure 6 also reveals that when the economy experiences a relative calm period with respect to the supply shocks, persistence of the inflation rate ($\phi_{1,t}$) and the importance of expectations in the determination of the inflation rate ($\phi_{2,t}$) are greater. This happens towards the end of the period of analysis. The trend is lower in the case of the response of inflation to the business cycle ($\phi_{3,t}$) and to variations in the real exchange rate ($\phi_{4,t}$). Contrary to this, when the supply shocks are highly volatile there is no a definite trend for the Phillips curve parameters.

Finally, parameters associated to the UIP (figure 7) show substantial differences depending on the state of shocks. There is no defined tendency in any of the cases. Moreover, in the entire period of analysis they are more stable in the low volatility state than in the high volatility state. In this latter state there are two defined periods: one that covers the decade of the 90s, during which the parameters showed greater stability, and another, from the year 2000 onward, in which the parameters increased substantially their volatility, as well as their magnitude in comparison with the first period. This could be explained by the adoption of a completely flexible exchange rate scheme in 1999. Also, the estimations suggest that the economy was experiencing a high volatility state of shocks in the entire period since the occurrence probability of this state did not fall below 0.7 at any time.

*Figure 5: Time-Varying Parameters Estimated for the Aggregate Demand*

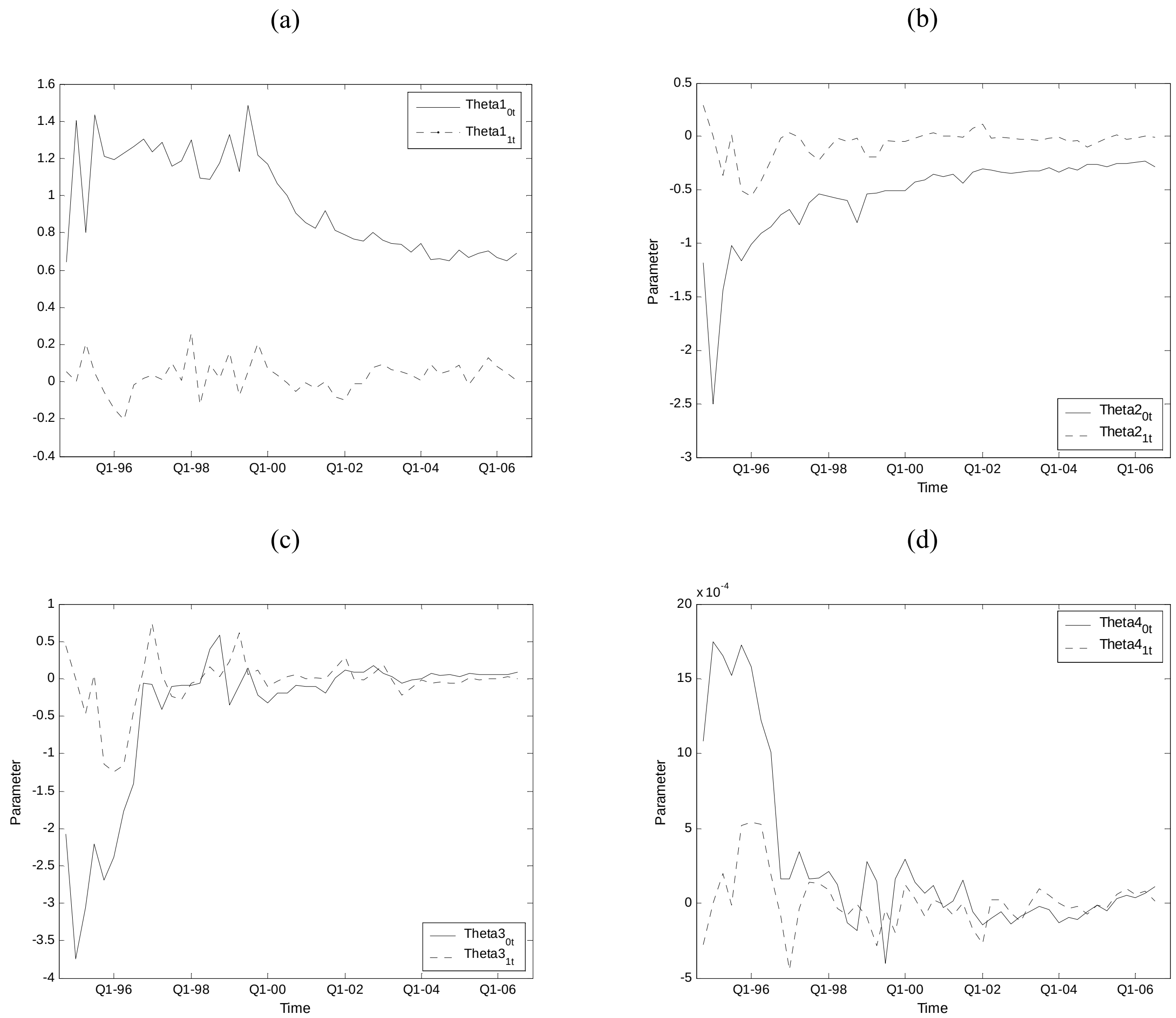


*Figure 6: Time-Varying Parameters Estimated for the Phillips Curve*

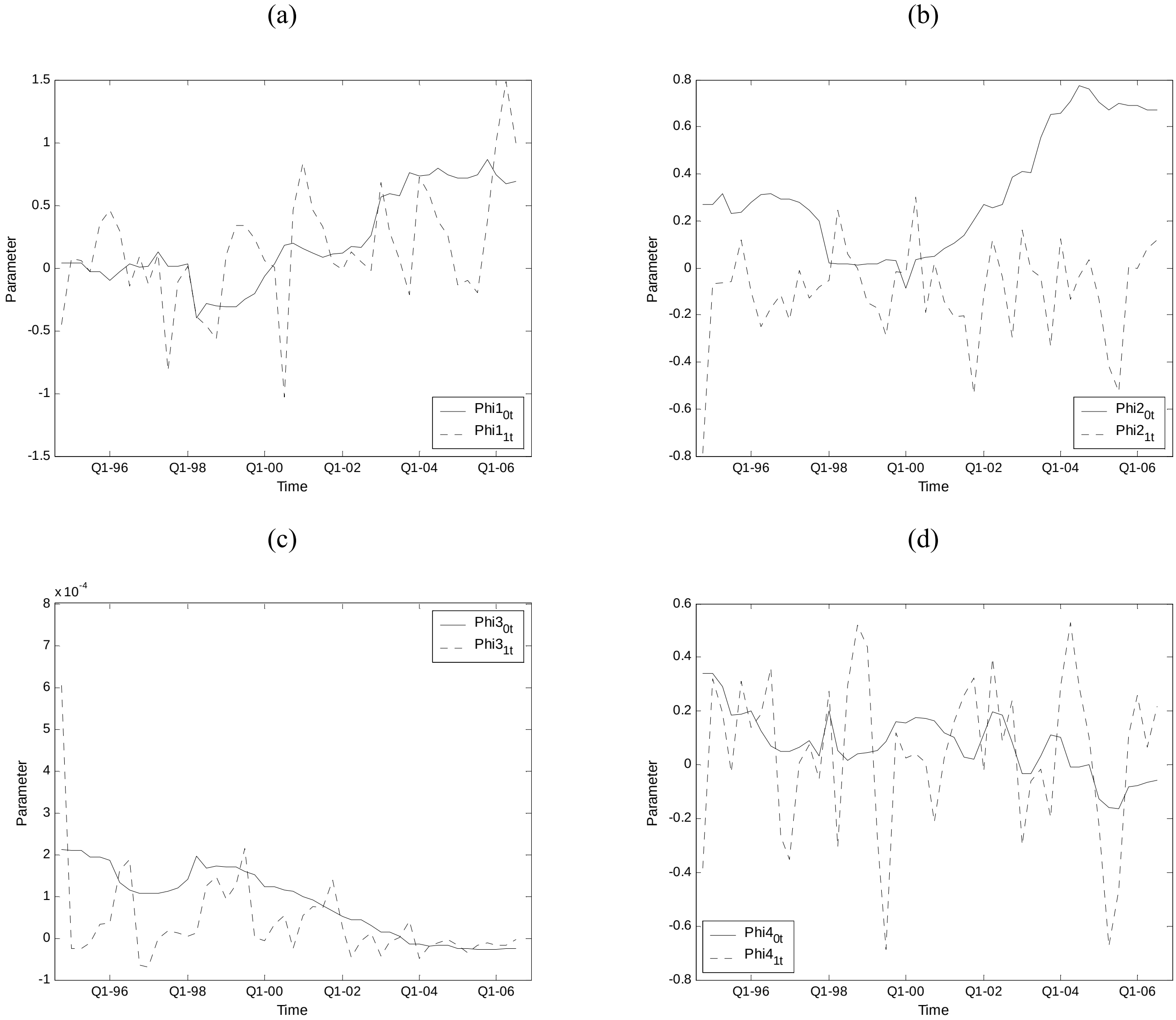
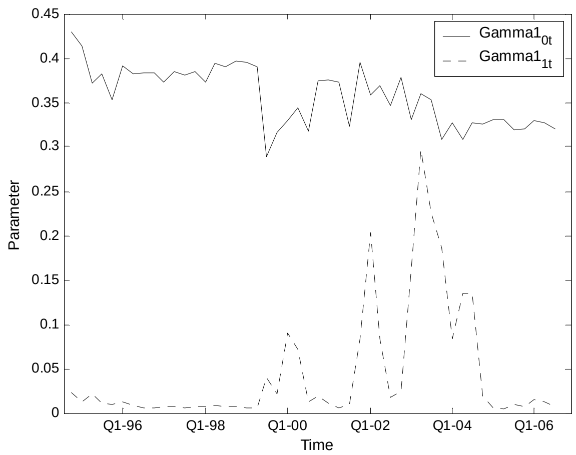
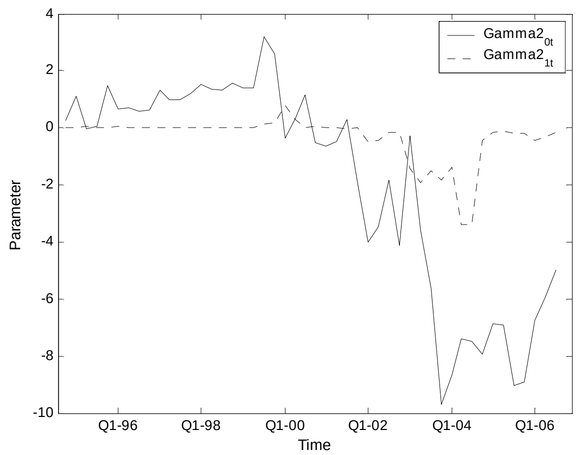


*Figure 7: Time-Varying Parameters Estimated for the Real Uncovered Interest Parity*

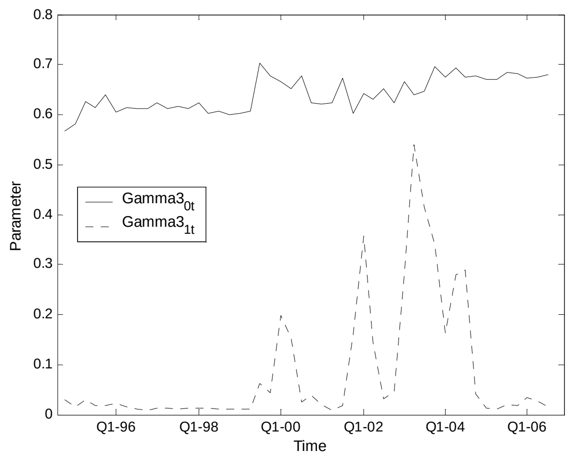
(a)




(b)




(c)




Based on the estimated parameters presented in table 5, we calculated the decomposition of the conditional variance of the forecast error. Figure 8 shows the decomposition for the set of equations associated with aggregate demand. Total uncertainty in the output gap (aggregate demand) equation has been relatively high throughout the entire period (the output gap is measured as the percentage deviation of output with respect to its trend). On average the forecast error variance was 0.021, of which 87.6% was explained by uncertainty in the demand shocks and 12.4% by instabilities in the model parameters (see table 6). Also, total uncertainty revealed significant spikes (almost twice the average) in the mid 90s and during the period of 1998-1999. However, after the year 2000, total uncertainty declined on average in a little over 30% with respect to the average observed between 1993 and 1999. Similar results are obtained with the contributions of additive and multiplicative uncertainty to total uncertainty. While parameter instability contributed approximately 15% to total uncertainty throughout the 90s, this contribution decreased to less than 10% in the period subsequent to the year 2000.

***Figure 8: Decomposition of the Conditional Variance of the Forecast Error of the Output Gap***

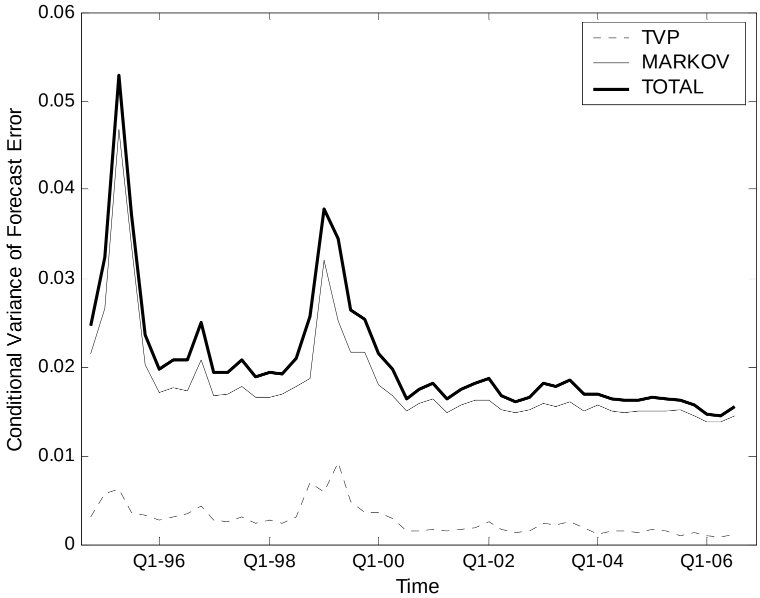



***Table 6: Decomposition of the Conditional Variance of the Forecast Error of the Output Gap***

| | Conditional Variance of the Forecast Error | | | Percentage | |
|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | MARKOV |
| 1993-1999 | 0.00407 | 0.02173 | 0.02580 | 15.7% | 84.3% |
| 2000-2006 | 0.00160 | 0.01535 | 0.01696 | 9.3% | 90.7% |
| Total Sample | 0.00279 | 0.01842 | 0.02121 | 12.4% | 87.6% |

The decomposition of the conditional variance of the forecast error for the inflation rate (Phillips curve) equation is shown in figure 9. Results in this case are similar to those found for the output gap with respect to magnitude and behavior (principally for the decade of the 90s). Total uncertainty associated to the inflation rate has been on average 0.015 for the entire period, of which 69.9% is explained by uncertainty in the supply shocks and 30.1% by parameter instability (see table 7). The two recurrent periods of high uncertainty, as in the case of the output gap, are in the mid 90s and during 1998-1999, where uncertainty reached levels more than twice the observed average for the entire period. Even though additive uncertainty explains a largest share of total uncertainty in the entire period, for a brief episode during the Asian crises the contribution pattern is reverted and it is uncertainty in the parameters that is most relevant. Total inflation uncertainty, as in the case of the output gap, has decreased over time, while the contribution of additive uncertainty increased with time.

***Figure 9: Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate***

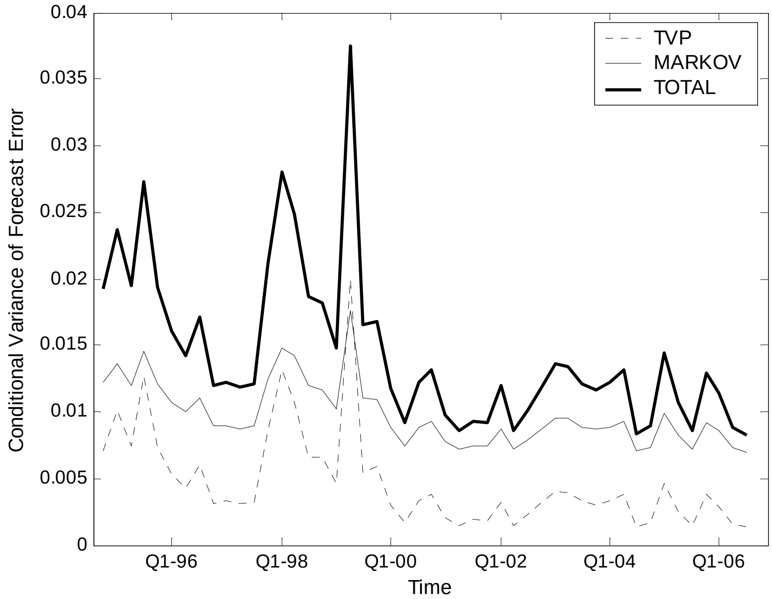



***Table 7: Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate***

| | Conditional Variance of the Forecast Error | | | Percentage | |
|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | MARKOV |
| 1993-1999 | 0.00852 | 0.01235 | 0.02087 | 37.5% | 62.5% |
| 2000-2006 | 0.00260 | 0.00818 | 0.01078 | 23.2% | 76.8% |
| Total Sample | 0.00545 | 0.01019 | 0.01563 | 30.1% | 69.9% |

Finally, figure 10 presents the decomposition of the conditional variance of the forecast error associated with the real exchange rate equation. Total uncertainty, measured by the variance, has been quite important throughout the period (approximately 4.1 on average) and basically explained (92%) by uncertainty in the shocks of the UIP or uncertainty in the risk premium that captures the effects of the unobservables of the exchange market sentiments. Also, total uncertainty has not shown a defined pattern over time (see table 8).

***Figure 10: Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate***

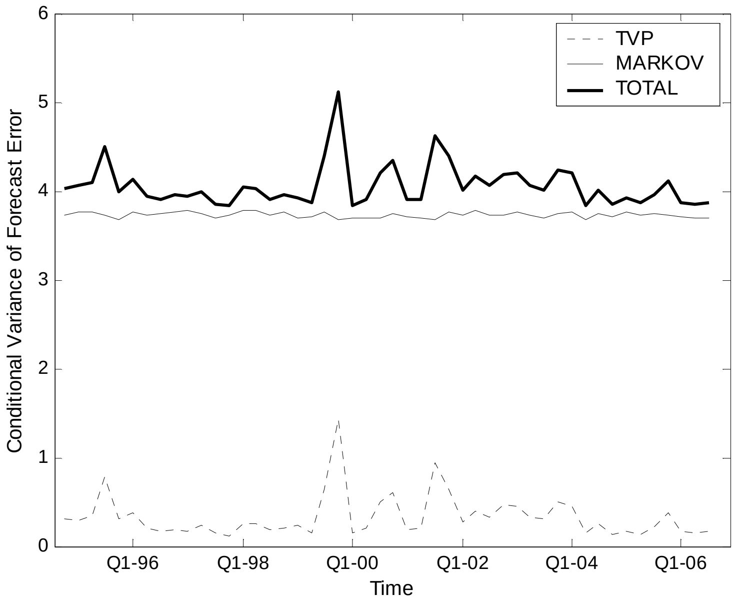



***Table 8: Decomposition of the Conditional Variance of the Forecast Error of the Real Exchange Rate***

| | Conditional Variance of the Forecast Error | | | Percentage | |
|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | MARKOV |
| 1993-1999 | 0.32816 | 3.73569 | 4.06385 | 8.8% | 92.3% |
| 2000-2006 | 0.32701 | 3.72663 | 4.05364 | 8.8% | 92.1% |
| Total Sample | 0.32756 | 3.73099 | 4.05855 | 8.8% | 92.2% |

Summing up, overall uncertainty is dominated by additive uncertainty in all three sets of equations (output gap, inflation and real exchange rate). Moreover, results of the estimations of the behavioral equations (aggregate demand and aggregate supply) suggest that the variance of shocks is state-dependent and that such states could be considered as high volatility periods in the shocks and low volatility periods. For these two sets of equations, total uncertainty has consistently declined during the current decade, bringing a rather long period of stability (so far) that coincides with the establishment of a *full-fledged inflation targeting* framework for the conduct of the Chilean monetary policy and an explicit rule for setting fiscal policy. On the other hand, in the 90s, total uncertainty showed substantial increases in the output gap and the inflation rate, identifying clearly the two states in the variance of shocks. This also indicates that during these periods the Chilean economy experienced a high volatility state of shocks. Finally, uncertainty in the real exchange rate is basically explained by the exchange market shocks and has not shown a decreasing pattern in time as in the case of inflation and the output gap.

We use the Bootstrap methodology to verify whether the differences between the variance of the forecast error due to additive uncertainty and that due to multiplicative uncertainty are statistically significant and whether the assumption of Gaussian errors in the estimation introduces important biases.[36] The most important findings of this exercise can be summarized as follows (for details on the results see appendix E): (1) even though the bootstrap average estimations and those based on the assumption of Gaussian errors differ, the bias does not seem to be important in magnitude; and (2) the bootstrap estimations confirm the observed trends in total uncertainty (figures 8 to 10), as well as the statistical significance of the differences in the decomposition of the variance.

To conclude this subsection we present a robustness analysis for the decomposition of the forecast error variance. In section 3 we found evidence of important differences in the estimation of the output gap when we consider five output detrending methods. Given that the aggregate demand and the Phillips curve equations contemplate an output gap measure for its estimation, measurement errors in the estimation of this variable will be part of the additive and multiplicative uncertainty without any possibility of discrimination.[37] Tables 9 and 10 show the results of the decomposition of uncertainty in its two sources, additive and multiplicative, for these two equations and for each of the five filters used in section 3. The

[36] The bootstrap re-sampling was done following the methodologies of Stoffer and Wall (1991) and Psaradakis (1998) for state-space models using the Kalman filter and for the sampling of errors with Markov regime changes, respectively.

[37] Recall that when the measurement error is associated to the dependent variable, as in the case of the aggregate demand, the estimated parameters will still be unbiased and consistent. The measurement error will be captured by the regression error. On the other hand, when the measurement error is associated with one or more independent variables, as is the case of the Phillips curve, the parameters will be biased and inconsistent. Even though the measurement error affects in different ways depending on the type of variable that it affects, this could have implications on the decomposition of uncertainty (through the error or the magnitude of the parameters).

first row of both tables show the decomposition presented in the analysis of this subsection, where the gap was calculated using the HP filter, and hence, represents our benchmark. In the case of the output gap (table 9) it is observed that in general total uncertainty is quite similar for all filters and that differences, as is expected, arises in the contribution of each type of uncertainty to total uncertainty. However, all detrending methods keep additive uncertainty as the most important source of uncertainty (its contribution varies from a minimum of 84.7% with the BK filter and a maximum of 90% with the Clark filter). With respect to the inflation rate (table 10) the difference between filters can be observed in both the estimation of total uncertainty and the contributions of each type of uncertainty to total uncertainty. In the former case, the estimations are in the range of 0.01374 and 0.02274 calculated using the BK filter and the quadratic trend, respectively, while the contributions of the additive uncertainty vary between 66.6% obtained using the BK filter and 73.5% using the Clark filter. It is important to highlight that in this case additive uncertainty also explains total uncertainty of inflation, regardless of the method considered for the estimation of the output gap. These results strengthen the conclusions mentioned before regarding the importance of additive uncertainty for the Chilean economy.

***Table 9: Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Output Gap***

| | Conditional Variance of the Forecast Error | | | Percentage | |
|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | MARKOV |
| Hodrick-Prescott | 0.00279 | 0.01842 | 0.02121 | 13.2% | 86.8% |
| Baxter-King | 0.00314 | 0.01734 | 0.02048 | 15.3% | 84.7% |
| Christiano-Fitzgerald | 0.00304 | 0.01733 | 0.02037 | 14.9% | 85.1% |
| Quadratic-Trend | 0.00287 | 0.01901 | 0.02189 | 13.1% | 86.9% |
| Clark | 0.00200 | 0.01803 | 0.02003 | 10.0% | 90.0% |

***Table 10: Robustness Analysis for the Decomposition of the Conditional Variance of the Forecast Error of the Inflation Rate***

| | Conditional Variance of the Forecast Error | | | Percentage | |
|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | MARKOV |
| Hodrick-Prescott | 0.00545 | 0.01019 | 0.01563 | 34.8% | 65.2% |
| Baxter-King | 0.00385 | 0.00988 | 0.01374 | 28.0% | 72.0% |
| Christiano-Fitzgerald | 0.00393 | 0.01006 | 0.01398 | 28.1% | 71.9% |
| Quadratic-Trend | 0.00761 | 0.01514 | 0.02274 | 33.4% | 66.6% |
| Clark | 0.00504 | 0.01397 | 0.01901 | 26.5% | 73.5% |

## 5. Final Remarks

Macroeconomic policy in Chile is currently of world class quality. The Central Bank of Chile has been operating within a full-fledged inflation targeting framework since 1999-2000 while fiscal policy has been bounded by an explicit budget rule that takes away pro-cyclical influences since 2001. As a result, inflation has remained within the inflation target range most of the time and economic activity has grown steadily between 2 and 6% annually (with no recessions nor booms whatsoever). This rather stable period also appears in our findings in the sense that overall uncertainty concerning monetary policy has declined in the first seven years of the current decade. It has also implied a greater role for uncertainty attributed to shocks (and less to uncertainty linked to unstable parameters) in both the cases of inflation and the output gap, as it could be expected. However, the prominence of additive uncertainty is a hallmark for the entire period, including both the tranquil first decade of the 21st century and the more volatile 90s. This means that investigating the (stochastic) nature of shocks affecting the Chilean economy should be high in the research agenda of the Central Bank.

The full-fledged inflation targeting scheme applied since 1999 came with a floating exchange rate and no explicit or implicit target for the exchange rate (as it was loosely the case during most of the 90s). This important policy innovation has left the exchange rate as the main adjustment variable – a sort of fuse –, a feature that shows in our results: parameters in the exchange rate equation are less stable in the current decade than they used to be in the 90s.

It is important to note our findings assume that there is no model uncertainty and, thus, the only uncertainties relevant for the conduct of monetary policy are those in the shocks and parameters. Hence, they should be interpreted with caution. To analyze uncertainty in the model it could be possible to estimate the behavioral equations of the economy using the methodology presented in this paper but using different specifications. With this, it could be verified whether the decomposition of the uncertainty found here holds or not.[38] We leave this exercise pending for future research.

Finally, results on uncertainty about the quality and completeness of output gap data reveal that, among other things, using the Hodrick-Prescott filter based on real time data could be misleading. Hence, the Central Bank of Chile should consider a wide spectrum of filters for detrending real activity data[39] and, more importantly, use an ample menu of proxy variables to check for the economy's temperature when making its monetary policy decisions. The literature suggests monetary policy rules more "immune" to

[38] This exercise was done only with the UIP under two specifications: the original equation of Svensson (2000) and Al-Eyd and Karasulu (2006) and the equation that includes the backward-looking term to allow deviations from the parity (presented here). We found that although the behavior of the parameters and the magnitude of total uncertainty change significantly, the decomposition of the uncertainty is not altered (additive uncertainty is maintained as the principal factor of uncertainty).

[39] And also use some alternative methodologies for estimating potential output, as it actually does,

this type of uncertainty that consider, for example, output growth rates or unemployment level rates (as opposed to the output gap).

# References

[1] Al-Eyd, Ali J. and Meral Karasulu. 2006. "Ambition versus Gradualism in Disinflation Horizons under Bounded Rationality: The Case of Chile." *International Monetary Found*, Working Paper.

[2] Balvers, Ronald and Thomas Cosimano. 1994. "Inflation Variability and Gradualist Monetary Policy." *Review of Economic Studies*, 61:4, pp. 721-38.

[3] Basar, Tamer and Mark Salomon. 1990. "Credibility and the Value of Information Transmission in a Model of Monetary Policy and Inflation." *Journal of Economic Dynamics & Control*, 14:1, pp. 97-116.

[4] Baxter, Marianne and Robert G. King. 1999. "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." *Review of Economics and Statistics*, 81:4, pp. 575-93.

[5] Beck, Gunter W. and Volker Wieland. 2002. "Learning and control in a changing economic environment." *Journal of Economic Dynamics and Control*, 26, pp. 1359-77.

[6] Bernhardsen, Tom, Oyvind Eitrheim, Anne Sofie Jore, and Oistein Roisland. 2005. "Real-Time Data for Norway: Challenges for Monetary Policy." *North American Journal of Economics and Finance*, 16:3, pp. 333-49.

[7] Bertocchi, Graziella and Michael Spagat. 1993. "Learning, Experimentation, and Monetary Policy." *Journal of Monetary Economics*, 32:1, pp. 169-83.

[8] Blinder, A. S. 1998. *Central Banking in Theory and Practice*, Cambridge: MIT Press.

[9] Brainard, William C. 1967. "Uncertainty and the Effectiveness of Policy." *The American Economic Review*, 57:2, pp. 411-25.

[10] Calvo, Guillermo. 1983. "Staggered Prices in a Utility Maximization Framework." *Journal of Monetary Economics*, 12:4, pp. 383-98.

[11] Caputo, Rodrigo, Felipe Liendo, and Juan Pablo Medina. 2006. "New Keynesian Models For Chile During The Inflation Targeting Regime: A Structural Approach." *Central Bank of Chile*, Working Paper 402.

[12] Céspedes, Luis F., Marcelo Ochoa, and Claudio Soto. 2005. "The New Keynesian Phillips Curve in an Emerging Market Economy: The Case of Chile." *Central Bank of Chile*, Working Paper 355.

[13] Clarida, Richard, Jordi Gali, and Mark Gertler. 1998. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *National Bureau of Economic Research*, Working Papers: 6442.

[14] Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, 37:4, pp. 1661-707.

[15] Clark, Peter K. 1987. "The Cyclical Component of U.S. Economic Activity." *Quarterly Journal of Economics*, 102:4, pp. 797-814.

[16] Cogley, Timothy and Thomas J. Sargent. 2001. "Evolving Post-World War II U.S. Inflation Dynamics," in *NBER macroeconomics annual 2001*. Ben Bernanke and Kenneth Rogoff eds: MIT Press, pp. 331-73.

[17] Cogley, Timothy and Thomas J. Sargent. 2005. "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US." *Review of Economic Dynamics*, 8:2, pp. 262-302.

[18] Corbo, Vittorio. 2002. "Monetary Policy in Latin America in the 90s " in *Monetary Policy: Rules and Transmission Mechanisms.* Norman Loayza and Klaus Schmidt-Hebbel eds.

[19] Craine, Roger. 1979. "Optimal Monetary Policy with Uncertainty." *Journal of Economic Dynamics and Control*, 1, pp. 59-83.

[20] Croushore, Dean and Tom Stark. 2001. "A Real-Time Data Set for Macroeconomists." *Journal of Econometrics*, 105, pp. 111-30.

[21] Christiano, Lawrence J. and Terry J. Fitzgerald. 2003. "The Band Pass Filter." *International Economic Review*, 44:2, pp. 435-65.

[22] Feldstein, Martin. 2003. "Monetary Policy in an Uncertain Environment." *National Bureau of Economic Research*, Working Papers: 9969.

[23] Fuentes, Rodrigo, Fabián Gredig, and Mauricio Larraín. 2007. "Estimating the Output Gap for Chile." *Central Bank of Chile*, Working Paper 455.

[24] Gali, Jordi and Mark Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics*, 44:2, pp. 195-222.

[25] Gali, Jordi, Mark Gertler, and J. David Lopez Salido. 2005. "Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve." *Journal of Monetary Economics*, 52:6, pp. 1107-18.

[26] Gallego, Francisco and Christian Johnson. 2001. "Teorías y Métodos de Medición del Producto de Tendencia: Una Aplicación al Caso de Chile " *Revista Economía Chilena*, 4:2.

[27] Ghysels, Eric, Norman R. Swanson, and Myles Callan. 2002. "Monetary Policy Rules with Model and Data Uncertainty." *Southern Economic Journal*, 69:2, pp. 239-65.

[28] Giannoni, Marc P. 2002. "Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy In A Forward-Looking Model." *Macroeconomic Dynamics*, 6, pp. 111-44.

[29] Gonzalez, Fidel and Arnulfo Rodriguez. 2003. "Robust control: A note on the response of the control to changes in the "free" parameter." *Mimeo*.

[30] Grauwe, Paul De and Marc-Alexandre Senegas. 2006. "Monetary policy design and transmission asymmetry in EMU: Does uncertainty matter?" *European Journal of Political Economy*, 22, pp. 787– 808.

[31] Gredig, Fabián. 2007. "Índices Podados como Medidas de Tendencia para el IMACEC." *Central Bank of Chile*, Working Paper 414.

[32] Hall, Simon, Chris Salmon, Tony Yates, and Nicoletta Batini. 1999. "Uncertainty and Simple Monetary Policy Rules - An illustration for the United Kingdom." *Bank of England*, Working Papers #96.

[33] Hamilton, James. 1990. "Analysis of Time Series Subject to Changes in Regime." *Journal of Econometrics*, 45.

[34] Hamilton, James 1989. "A New Approach to the Economic Analysis of Non-Stationary Time Series and The Bussines Cycles." *Econometrica*, 57:2.

[35] Hodrick, Robert J. and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit & Banking*, 29:1, pp. 1-16.

[36] Holt, Charles C. 1962. "Linear Decision Rules for Economic Stabilization and Growth." *Quarterly Journal of Economics*, 76:1, pp. 20-45.

[37] Holly, Sean and Andrew Hughes Hallet. 1989. *Optimal Control, Expectations and Uncertainty*: Cambridge University Press.

[38] Isard, Peter, Douglas Laxton, and Ann Charlotte Eliasson. 1999. "Simple Monetary Policy Rules under Model Uncertainty." *International Tax and Public Finance*, 6:4, pp. 537-77.

[39] Kim, Chang Jin. 1993. "Sources of Monetary Growth Uncertainty and Economic Activity: The Time-Varying-Parameter Model with Heteroskedastic Disturbances." *Review of Economics and Statistics*, 75:3, pp. 483-92.

[40] Kim, Chang Jin. 2004. "Markov-Switching Models with Endogenous Explanatory Variables." *Journal of Econometrics*, 122:1, pp. 127-36.

[41] Kim, Chang Jin. 2006. "Time-Varying Parameter Models with Endogenous Regressors." *Economics Letters*, 91:1, pp. 21-26.

[42] Kim, Chang Jin and Charles R. Nelson. 1999. *State-space models with regime switching: Classical and Gibbs-sampling approaches with applications*, Cambridge: MIT Press.

[43] Kim, Chang Jin and Charles R. Nelson. 2006. "Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model Using ex post Data." *Journal of Monetary Economics*, 53:8, pp. 1949-66.

[44] Kozicki, Sharon. 2004. "How do data revisions affect the evaluation and conduct of monetary policy?" *Federal Reserve Bank of Kansas, Economic Review*, first quarter.

[45] Kuttner, Kenneth N. 1994. "Estimating Potential Output as a Latent Variable." *Journal of Business and Economic Statistics*, 12:3, pp. 361-68.

[46] Martin, Ben and Chris Salmon. 1999. "Should uncertain monetary policy-makers do less?" *Bank of England*, Working Papers #99.

[47] Meyer, Laurence H., Eric T. Swanson, and Volker W. Wieland. 2001. "NAIRU Uncertainty and Nonlinear Policy Rules " *The American Economic Review*, 92:2, pp. 226-31.

[48] Onatski, Alexei and James H. Stock. 2002. "Robust Monetary Policy Under Model Uncertainty In A Small Model Of The U.S. Economy." *Macroeconomic Dynamics*, 6, pp. 85-110.

[49] Orphanides, A, R. D. Porter, D. Reifschneider, R. J. Tetlow, and F. Finan. 2000. "Errors in the Measurement of the Output Gap and the Design of Monetary Policy." *Journal of Economics and Business*, 52, pp. 117-41.

[50] Orphanides, Athanasios. 2001. "Monetary Policy Rules Based on Real-Time Data " *The American Economic Review*, 91:4, pp. 964-85.

[51] Orphanides, Athanasios and Simon van Norden. 1999. "The Reliability of Output Gap Estimates in Real Time." *Mimeo*.

[52] Orphanides, Athanasios and John C. Williams. 2002. "Imperfect Knowledge, Inflation Expectations, and Monetary Policy." *Federal Reserve Bank of San Francisco*, Working Paper 2002-04.

[53] Phillips, A. W. 1954. "Stabilisation Policy in a Closed Economy." *The Economic Journal*, 64:254, pp. 290-323.

[54] Psaradakis, Zacharias. 1998. "Bootstrap-based evaluation of markov-switching time series models." *Econometric Reviews*, 17:3, pp. 275-88.

[55] Roberts, John M. 1995. "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit & Banking*, 27:4, pp. 975-84.

[56] Roberts, John M. 2001. "How well does the new Keynesian sticky-price model fit the data?" *Board of Governors of the Federal Reserve System (U.S.)*, Finance and Economics Discussion Series: 2001-13.

[57] Rudebusch, Glenn D. 2001. "Is the Fed Too Timid? Monetary Policy in an Uncertain World." *The Review of Economics and Statistics*, 83:2, pp. 203-17.

[58] Sack, Brian. 2000. "Does the fed act gradually? A VAR analysis." *Journal of Monetary Economics*, 46, pp. 229-56.

[59] Sargent, Thomas J. 1998. "Discussion of ‘Policy Rules for Open Economies’ by Laurence Ball." *NBER Conference on ‘Monetary Policy Rules.*.

[60] Schellekens, Philip. 2002. "Caution and Conservatism in the Making of Monetary Policy." *Journal of Money, Credit, and Banking*, 34:1, pp. 160-77.

[61] Semmler, Willi, A. Greiner, and Wenlang Zhang. 2005. *Monetary and Fiscal Policies in the Euro-Area: Macro modeling, learning and empirics*. Amsterdam: Elsevier Science Publisher.

[62] Sims, Christopher A. 2001. "Evolving Post-World War II U.S. Inflation Dynamics: Comment," in *NBER macroeconomics annual 2001*. Ben Bernanke and Kenneth Rogoff eds: MIT Press, pp. 373-79.

[63] Sims, Christopher A. and Tao Zha. 2006. "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review*, 96:1, pp. 54-81.

[64] Söderström, Ulf. 2002. "Monetary Policy with Uncertain Parameters " *Scandinavian Journal of Economics*, 14:1, pp. 125-45.

[65] St-Amant, Pierre and Simon van Norden. 1998. "Measurement of the Output Gap: A discussion of recent research at the Bank of Canada." *Bank of Canada*, Technical Report, 79.

[66] Stoffer, David S. and Kent D. Wall. 1991. "Bootstapping State-Space Models: Gaussian Maximum Likelihood Estimation and the Kalman Filter." *Journal of the American Statistical Association*, 86:416, pp. 1024-33.

[67] Svensson, Lars E. O. 1997. "Inflation forecast targeting: Implementing and monitoring inflation targets " *European Economic Review*, 41:6, pp. 1111-46.

[68] Svensson, Lars E. O. 1999. "Inflation Targeting: Some Extensions " *Scandinavian Journal of Economics*, 101:3, pp. 337-61.

[69] Svensson, Lars E. O. 2000. "Open-Economy Inflation Targeting." *Journal of International Economics*, 50:1, pp. 155-83.

[70] Tetlow, Robert J. and Peter von sur Muehlen. 2001. "Simplicity Versus Optimality: The Choice of Monetary Policy Rules when Agents Must Learn." *Journal of Economic Dynamics and Control*, 25, pp. 245-79.

[71] Theil, Henri. 1964. *Optimal Decision Rules for Government and Industry*, Rand McNally.

[72] Walsh, Carl E. 2003. "Implications of a Changing Economic Structure for the Strategy of Monetary Policy." *Mimeo*.

[73] Wieland, Volker. 2000. "Monetary policy, parameter uncertainty and optimal learning " *Journal of Monetary Economics*, 46:1, pp. 199-228.

[74] Zampolli, Fabrizio. 2006. "Optimal monetary policy in a regime-switching economy: The response to abrupt shifts in exchange rate dynamics." *Journal of Economic Dynamics & Control*, 30, pp. 1527–67.

[75] Zhang, Wenlang and Willi Semmler. 2005. "Monetary Policy Rules under Uncertainty: Empirical Evidence, Adaptive Learning, and Robust Control." *Macroeconomic Dynamics*, 9:5, pp. 651-81.

## Appendix A: Robustness Test for Reliability of Real-Time Estimates using Non-Seasonally Adjusted Data and Seasonal Dummies.

In the following tables we present the detail of the results obtained in the estimation of the output gap with real-time data using non-seasonally adjusted data and seasonally adjusted data through seasonal dummy variables.

***Descriptive Statistics of the Total Revisions in the Output Gap***
***(Using Non-Seasonally Adjusted Data)***

| | **Mean** | **Std** | **Min** | **Max** | **AR** |
|---|---|---|---|---|---|
| Hodrick-Prescott | -0.005 | 0.015 | -0.036 | 0.031 | 0.331 |
| Baxter-King | 0.006 | 0.007 | -0.008 | 0.023 | 0.722 |
| Christiano-Fitzgerald | -0.013 | 0.009 | -0.029 | 0.005 | 0.836 |
| Quadratic-Trend | -0.011 | 0.021 | -0.050 | 0.033 | 0.676 |
| Clark | 0.001 | 0.006 | -0.014 | 0.010 | 0.023 |

***Descriptive Statistics of the Reliability Measures for the Alternative Distinct Filters***
***(Using Non-Seasonally Adjusted Data)***

| | **Corr** | **N/S** | **Opsign** | **Xsize** |
|---|---|---|---|---|
| Hodrick-Prescott | 0.773 | 0.754 | 0.286 | 0.536 |
| Baxter-King | 0.529 | 0.958 | 0.286 | 0.464 |
| Christiano-Fitzgerald | 0.244 | 1.290 | 0.393 | 0.821 |
| Quadratic-Trend | 0.846 | 0.642 | 0.179 | 0.393 |
| Clark | 0.963 | 0.290 | 0.036 | 0.107 |

***Descriptive Statistics of the Total Revisions in the Output Gap***

***(Using Seasonal Dummies)***

| | Mean | Std | Min | Max | AR |
|---|---|---|---|---|---|
| Hodrick-Prescott | 0.002 | 0.017 | -0.034 | 0.031 | 0.260 |
| Baxter-King | 0.008 | 0.007 | -0.002 | 0.019 | 0.874 |
| Christiano-Fitzgerald | -0.011 | 0.010 | -0.029 | 0.002 | 0.942 |
| Quadratic-Trend | -0.004 | 0.024 | -0.051 | 0.046 | 0.521 |
| Clark | 0.005 | 0.007 | -0.013 | 0.017 | -0.063 |

***Descriptive Statistics of the Reliability Measures for the Alternative Distinct Filters***

***(Using Seasonal Dummies)***

| | Corr | N/S | Opsign | Xsize |
|---|---|---|---|---|
| Hodrick-Prescott | 0.413 | 1.044 | 0.321 | 0.429 |
| Baxter-King | 0.646 | 0.772 | 0.321 | 0.500 |
| Christiano-Fitzgerald | 0.312 | 1.031 | 0.357 | 0.571 |
| Quadratic-Trend | 0.745 | 0.771 | 0.179 | 0.321 |
| Clark | 0.932 | 0.367 | 0.071 | 0.214 |

# Appendix B: Estimation based on the Kalman filter and the EM algorithm (Kim and Nelson, 1999)

**1. Kalman Filter**

$\beta_{t|t-1}^{(i,j)}, P_{t|t-1}^{(i,j)}, \tau_{t|t-1}^{(i,j)}, f_{t|t-1}^{(i,j)}, H_{t|t-1}^{(i,j)}$

**2. Hamilton's EM Algorithm**

$$\Pr(S_t, S_{t-1} \mid \psi_{t-1}) = \Pr(S_t, S_{t-1}) \Pr(S_{t-1} \mid \psi_{t-1})$$

$$f(x_t \mid \psi_{t-1}) = \sum_{S_t} \sum_{S_{t-1}} f(x_t \mid S_t, S_{t-1}, \psi_{t-1}) \Pr(S_t, S_{t-1} \mid \psi_{t-1})$$

$$l(\theta) = l(\theta) + \ln(f(y_t \mid \psi_{t-1}))$$

$$\Pr(S_t, S_{t-1} \mid \psi_{t-1}) = \frac{f(x_t, S_t, S_{t-1}, \psi_{t-1})}{f(x_t \mid \psi_{t-1})} = \frac{f(x_t \mid S_t, S_{t-1}, \psi_{t-1}) \Pr(S_t, S_{t-1} \mid \psi_{t-1})}{f(x_t \mid \psi_{t-1})}$$

$$\Pr(S_t \mid \psi_t) = \sum_{S_{t-1}} \Pr(S_t, S_{t-1} \mid \psi_t)$$

**3. Approximations**

$$\beta_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_{t-1} = i, S_t = j \mid \psi_t) \beta_{t|t}^{(i,j)}}{\Pr(S_t = j \mid \psi_t)}$$

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_{t-1} = i, S_t = j \mid \psi_t) \left\{ P_{t|t}^{(i,j)} + (\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)})(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)})' \right\}}{\Pr(S_t = j \mid \psi_t)}$$

**4. Loglikelihood function**

$$l(\theta) = \sum_{t=1}^{T} \ln(f(x_t \mid \psi_{t-1}))$$

## Appendix C: Kalman filter with endogenous regressors (Kim, 2006)

$$\beta_{t|t-1} = E(\beta_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = \beta_{t-1|t-1}$$

$$P_{t|t-1} = Var(\beta_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = P_{t-1|t-1} + Q_\eta$$

$$\tau_{t|t-1} = x_t - E(x_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = x_t - (w_t, v_t)'\beta_{t-1|t-1} - \xi_t^{*}{}'\rho\sigma_\varepsilon$$

$$H_{t|t-1} = Var(x_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = (w_t, v_t)'P_{t|t-1}(w_t, v_t) + (1-\rho'\rho)\sigma_\varepsilon^2$$

$$\beta_{t|t} = E(\beta_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = \beta_{t|t-1} + P_{t|t-1}(w_t, v_t)H_{t|t-1}^{-1}\tau_{t|t-1}$$

$$P_{t|t} = Var(\beta_t \mid w_t, v_t, \xi_t^*, \psi_{t-1}) = P_{t|t-1} - P_{t|t-1}(w_t, v_t)H_{t|t-1}^{-1}(w_t, v_t)'P_{t|t-1}$$

## Appendix D: Loglikelihood function (Kim and Nelson, 1999)

$$\begin{aligned} f(x_t \mid \psi_{t-1}) &= \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_t, S_t = i, S_{t-1} = j \mid \psi_{t-1}) \\ &= \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_t \mid S_t = i, S_{t-1} = j \mid \psi_{t-1}) \Pr[S_t = i, S_{t-1} = j \mid \psi_{t-1}] \end{aligned}$$

where:

$$f(x_t \mid S_t = i, S_{t-1} = j, \psi_{t-1}) = (2\pi)^{-\frac{N}{2}} \mid f_{t|t-1}^{(i,j)} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\tau_{t|t-1}^{(i,j)}{}' f_{t|t-1}^{(i,j)^{-1}} \tau_{t|t-1}^{(i,j)}\right\}$$

# Appendix E: Bootstrap of the decomposition of the conditional variance of the forecast error.

In the following Table we present the results obtained from the bootstrap of the decomposition of the conditional variance of the forecast error for the three models (mean estimation and 95% confidence intervals). Additionally, in the same table we present, for comparison purposes, the results found before under the assumption of the Gaussian errors in the estimation. The bootstrap re-sampling was done following the methodologies of Stoffer and Wall (1991) and Psaradakis (1998) for state-space models that use the Kalman filter and for the sampling of errors with Markov regime changes, respectively.

***Bootstrap Decomposition of the Conditional Variance of the Forecast Error***

| | Gaussian ML | | | Bootstrap | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TVP | MARKOV | TOTAL | TVP | | | MARKOV | | | TOTAL | | |
| | | | | Mean | [ CI 95% | ] | Mean | [ CI 95% | ] | Mean | [ CI 95% | ] |
| Output Gap | | | | | | | | | | | | |
| 1993-1995 | 0.00424 | 0.02566 | 0.02990 | 0.00585 | 0.00572 | 0.00598 | 0.05667 | 0.05542 | 0.05790 | 0.06251 | 0.06119 | 0.06384 |
| 1996-1998 | 0.00353 | 0.01881 | 0.02234 | 0.00548 | 0.00533 | 0.00564 | 0.02330 | 0.02264 | 0.02401 | 0.02878 | 0.02796 | 0.02961 |
| 1999-2006 | 0.00208 | 0.01616 | 0.01824 | 0.00193 | 0.00188 | 0.00197 | 0.01807 | 0.01749 | 0.01870 | 0.02000 | 0.01938 | 0.02066 |
| Total Sample | 0.00279 | 0.01842 | 0.02121 | 0.00342 | 0.00334 | 0.00351 | 0.02596 | 0.02524 | 0.02671 | 0.02938 | 0.02860 | 0.03020 |
| Inflation Rate | | | | | | | | | | | | |
| 1993-1995 | 0.01172 | 0.01428 | 0.02599 | 0.01204 | 0.01199 | 0.01207 | 0.06555 | 0.02062 | 0.15871 | 0.07758 | 0.03267 | 0.18638 |
| 1996-1998 | 0.00612 | 0.01099 | 0.01711 | 0.00588 | 0.00586 | 0.00590 | 0.04010 | 0.01541 | 0.09914 | 0.04598 | 0.02129 | 0.09725 |
| 1999-2006 | 0.00337 | 0.00869 | 0.01205 | 0.00289 | 0.00288 | 0.00291 | 0.02381 | 0.01130 | 0.04950 | 0.02670 | 0.01420 | 0.05276 |
| Total Sample | 0.00545 | 0.01019 | 0.01563 | 0.00516 | 0.00514 | 0.00518 | 0.03479 | 0.01386 | 0.07986 | 0.03996 | 0.01903 | 0.08616 |
| Real Exchange Rate | | | | | | | | | | | | |
| 1993-1995 | 0.54577 | 10.54444 | 11.09022 | 0.81944 | 0.77777 | 0.86192 | 9.21296 | 9.02628 | 9.40670 | 10.03222 | 9.83458 | 10.24011 |
| 1996-1998 | 0.37324 | 10.46275 | 10.83625 | 0.74982 | 0.70561 | 0.79519 | 9.20454 | 9.01600 | 9.39305 | 9.95438 | 9.75733 | 10.16217 |
| 1999-2006 | 0.56164 | 10.58610 | 11.14777 | 0.91215 | 0.86287 | 0.96336 | 9.19028 | 8.99863 | 9.38625 | 10.10250 | 9.89786 | 10.31116 |
| Total Sample | 0.51542 | 10.55042 | 11.06592 | 0.85864 | 0.81185 | 0.90699 | 9.19750 | 9.00743 | 9.39136 | 10.05616 | 9.85448 | 10.26448 |