

## Top income measurement and undistributed profits

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## **Abstract**

Using retained profits, instead of realized capital gains, by simply adding them to the shareholders' other sources of income is shown to be in general incorrect. We provide a methodology to include undistributed profits as part of the income of the top echelons of the distribution.

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# Top income measurement and undistributed profits<sup>3</sup>

## 1. Introduction

In an effort to provide more accurate estimates of the distribution of income, recent literature has greatly focused on the measurement of top incomes and their participation on the national income (Atkinson et al., 2011; Alvaredo et al., 2013; Alvaredo and Londoño, 2013; Burdin et al., 2014, among others).<sup>4</sup> Most previous studies have considered only labor and other personal incomes excluding capital gains. More recently, a few studies have explicitly considered realized capital gains as part of the total income of the rich, showing that the top brackets of the distribution greatly increase their share in national income<sup>5</sup>. However, the use of realized capital gains has been criticized because they may reflect capital appreciations that take place over many years before the period in which the incomes are actually being measured (Armour et al, 2012; Smeeding and Thompson, 2010).

In view of this, other authors have used retained profits, instead of realized capital gains, by simply adding undistributed profits to the shareholders' other sources of income (Fairfield and Jorratt, 2014; López et al, 2013). This note shows that this procedure is in general incorrect and it also provides an approach that allows transforming retained profits into accrued capital gains which, in turn, can be directly added to other sources of income. In addition, we show the special conditions under which the approach of directly adding retained profits to the other income sources is correct. Moreover, we also

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<sup>4</sup> In this respect, Alvaredo (2011) has provided important insights by relating top incomes to the measurement of the Gini coefficient, the best known measure of inequality.

<sup>5</sup> For a complete discussion, see Atkinson et al., 2011

determine the conditions under which the new measure of income that includes regular income plus accrued capital gains distributes Pareto, and hence the Pareto interpolation can be used.

## 2. From undistributed profits to accrued capital gains

We use the measure of income developed by Haig (1921) and Simons (1938) where personal income of individual  $i$  at time  $t$  ( $I(t)$ ) is equal to her/his consumption ( $C(t)$ ) plus the net change on wealth ( $\Delta NW(t)$ ) (to simplify notation we drop subscript  $i$  for the individual):

$$I(t) = C(t) + \Delta NW(t) \quad (1)$$

One may alternatively define  $I(t)$  as the sum of the personal income  $y(t)$ , defined as the sum of labor income, distributed dividends and other personal incomes, plus accrued capital gains,  $G(t)$ , at time<sup>6</sup>  $t$ .

$$I(t) = y(t) + G(t) \quad (2)$$

In an economy subject to a variety of taxes the value of a dollar of retained profits by a firm cannot be directly attributed to the accrued income of the stockholder from such firm. That is, the implicit capital gain caused by retaining one dollar in the firm does not directly translate into one additional dollar of accrued capital gain.

Define, the opportunity cost of a retained dollar in terms of foregone dividend as,

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<sup>6</sup>  $I(t) = C(t) + \Delta NW(t) - G(t) + G(t)$  define  $y(t) = C(t) + \Delta NW(t) - G(t)$ , then we have  $I(t) = y(t) + G(t)$ .

$$\theta \equiv \frac{(1-\tau+s\tau)[1-m]}{(1-\tau)(1-z)} \quad (3)$$

Where  $\tau$  is the tax rate on firms' profits,  $m(y(t))$  is the personal tax rate on dividends,  $z$  is the tax rate on capital gains, and  $0 \leq s \leq 1$  is the fraction of the tax paid by the firm that is allowed as a tax credit to the stockholder. This formula is a slight generalization of the well-known formula developed by King (1974); unlike King's formulation, which assumes either no or complete tax integration (i.e.,  $s = 0$  or  $s = 1$ ), our specification allows for the existence of partial or total integration measured by the parameter  $s$ . Also, it is assumed that the tax rate on dividends is non-decreasing in  $y(t)$ .

The following proposition shows the relationship between retained or undistributed profits ( $\pi_r$ ) and accrued capital gains,

**Proposition 1.** *In equilibrium, accrued capital gains are related to retained profits as follows,*

$$G(t) = \theta(\tau, m(y), z; s)\pi_r(t), \quad (4)$$

**Proof:** *Equilibrium in the capital market implies (King, 1974):*

$$rV(t) = d(t) + (1 - z)(V(t + 1) - V(t)) \quad (5)$$

where  $V(t)$  is the value of the firm in time  $t$  and  $d(t)$  is the net after tax dividend paid by the firm.

We generalize (5) to allow for different degrees of tax integration ( $s$ ),

$$\frac{1-\tau+s\tau}{1-\tau}(1-m)rV(t) = d(t) + (V(t + 1) - V(t))(1 - z) \quad (6)$$

Using the definition of  $\theta$  in equation (3), (6) can be written as:

$$\theta rV(\tau) = \frac{d(t)}{1-z} + (V(t+1) - V(t)) \quad (7)$$

Using that  $d(t) = \theta((1 - \tau)\pi - \pi_r)$  and noting that in equilibrium  $rV(\tau) = (1 - \tau)\pi$ , we have,

$$G \equiv V(t+1) - V(t) \quad (8)$$

Then, we obtain that:

$$G(t) = \theta\pi_r \quad (9)$$

Thus, using equations (2) and (9) we obtain that total income is,

$$I(t) = y(t) + \theta(\tau, m(y(t)), z; s)\pi_r(t) \quad (10)$$

The following corollary follows,

**Corollary1.1.** *Only in the special case where  $\theta = 1$  it is legitimate to simple add on retained profits to the other incomes of shareholders to estimate their true total income.*

**Proof:** *Follows directly from Proposition 1.*

Thus, the approach of directly adding up retained profits to other income sources to estimate incomes of shareholders would be appropriate if the non-corporate tax system is neutral and if there is no tax integration. That is, when  $m = z$  and  $s = 0$ .

However, under most tax regimes  $\theta \neq 1$ . If  $\theta > 1$  it means that the opportunity cost of one dollar of foregone dividend is greater than one dollar, implying that firms would have incentives to distribute all their profits and hence that  $\pi_r = 0$ , in which case the problem would tend to disappear, at least in long run equilibrium. In practice, however, firms often do retain parts of their profits even in countries where  $\theta > 1$ . The case where  $\theta < 1$  is the most interesting one

because one can expect that in this case firms will have incentives to distribute the minimum possible dividends and, hence, retained profits are likely to be important. The opportunity cost of one dollar of foregone dividend is less than one dollar, which implies that firms will pay either zero dividends or the minimum level required by law. Thus, retained profits will likely be large and hence the incomes measured by simply adding the retained profits to the rest of the shareholders incomes will over estimate their true incomes.

### **Pareto distribution of accrued capital gains**

The remaining issue is whether or not  $\theta(\tau, m(y), z; s)\pi_r$  (accrued capital gains) distributes Pareto. Even if  $\pi_r$  distributes Pareto there is no guarantee that  $\theta(\tau, m(y), z; s)\pi_r$  distributes according to the same distribution. The main reason for this is that  $\theta$  is not constant as it is dependent on  $y(t)$  itself. Even if  $\theta$  is distributed Pareto, the product of two Pareto distributions is not necessarily Pareto. However, since we are dealing with top incomes one might assume that  $m(y) = m^{\max}$ ; that is,  $m$  is equal to the highest personal income marginal tax rate, in which case  $\theta$  is constant and independent of  $y(t)$ .

The following lemma provides the conditions under which accrued capital gains distribute Pareto.

**Lemma 1.** *Assume that  $m(y) = m^{\max}$ , with  $m^{\max} \geq m_i$  for all  $i$ , and that  $z$  is constant independent of the shareholder's income and that  $\pi_r$  is ruled by a regular varying distribution. Then  $\theta(\tau, m^{\max}, z; s)\pi_r$  also has a regular varying distribution and, moreover, it distributes Pareto.*

**Proof:** Since  $\theta$  is constant then  $\theta(\tau, m^{max}, z; s)\pi_r$  distributes according to a varying distribution given that  $\pi_r$  is ruled by to a regular varying distribution. Hence, given that all varying distributions are also Pareto we have that  $\theta(\tau, m^{max}, z; s)\pi_r$  has a Pareto distribution.

## Conclusion

An important practical implication of the analysis above is that measures using Pareto interpolation of top incomes (including accrued capital gains) should be restricted to the very top levels of the distribution, perhaps the top 0.1% and 0.01% richest segments of the population. This is so mainly because for lower income levels the marginal personal tax rate is likely to be below the maximum rate, and therefore  $m$  would be a nonlinear function of income and implying that income might not be distributed Pareto.

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