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## CONVERGENCE AND GROWTH: REVISITED

**Autor:** *José Miguel Benavente H.*  
*Emerson Melo*  
*Sandra Quijada J.*

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*Emerson Melo S.*  
Departamento de Economía  
Universidad de Chile

*José Miguel Benavente H.*  
Departamento de Economía  
Universidad de Chile

*Sandra Quijada J.*  
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**Abstract**

*The present paper builds on the work of Caselli, Esquivel and Lefort (1996) where growth equations are estimated using the Generalized Moment Method. It is shown here that the results of those authors are biased due to a methodological problem. The convergence rate of around 12% that they report is overestimated and the real rate is in fact around 3 or 4 %; in line with earlier studies. The results found are robust to various new dynamic panel estimation techniques, although the significant differences are indicated when making inferences if variance corrections are not considered for estimators that use the Generalized Moment Method.*

**Key words:** *Convergence, GMM, Kiviet Correction.*

**Resumen**

*El trabajo actual se apoya en el trabajo de Caselli, Esquivel y Lefort (1996) donde las growth equations son calculadas usando the Generalized Moment Method. Es mostrado que aquí que los resultados de esos escritores son parciales debido a un problema metodológico. La tasa de convergencia próxima a 12 % informada por los autores es sobreestimado y la tasa real es, de hecho, en torno de 3 o 4 %; en concordancia con estudios más tempranos. Los resultados encontrados son robustos para new dynamic panel estimation techniques, although the significant differences are indicated when making inferences if variance corrections are not considered for estimators that use the Generalized Moment Method.*

# Convergence and Growth: Revisited\*

José Miguel Benavente H.      Emerson Melo S.  
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## Resumen

The present paper builds on the work of Caselli, Esquivel and Lefort (1996) where growth equations are estimated using the Generalized Moment Method. It is shown here that the results of those authors are biased due to a methodological problem. The convergence rate of around 12 % that they report is overestimated and the real rate is in fact around 3 or 4 %; in line with earlier studies. The results found are robust to various new dynamic panel estimation techniques, although the significant differences are indicated when making inferences if variance corrections are not considered for estimators that use the Generalized Moment Method.

**Key words:** Convergence, GMM, Kiviet Correction.

## 1. Introduction

One of the most important neoclassical growth implications is that countries converge to a steady state growth condition. The empirical testing of this hypothesis has only offered evidence of conditional convergence; meanwhile absolute convergence has been rejected. Nevertheless, the debate remains open, as

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new estimation techniques suggest that earlier results may change significantly.

One of the most notable works in this respect has been that of Caselli, Esquivel and Lefort (1996), who, by employing dynamic panels, found the convergence rate to be around 10 %, as opposed to the 2 % approximate rate found by Barro and Sala-i-Martin previously. This result is due to the use of new econometric techniques which are expected to solve endogeneity problems, omitted variable and inconsistency of estimator bias.

The main objective of the present study is to test the evidence given by Caselli, Esquivel and Lefort (1996) for robustness by applying new dynamic panel estimation methods. In other words, we seek to discover if the 10 % convergence rate found by these authors holds. If this were the case, the result that, with a 10 % rate, countries would habitually find themselves near their steady state would become more robust.

It is shown here that the results of those authors are biased due to a methodological problem. The convergence rate of around 12 % that they report is overestimated and the real rate is in fact around 3 or 4 %; in line with earlier studies. The results found are robust to various new dynamic panel estimation techniques, although the significant differences are indicated when making inferences if variance corrections are not considered for estimators that use the Generalized Moment Method.

Concordantly, the present study is organized as follows: There is a brief review of growth literature in section two; the methodology and data used is presented in section three; the results are presented in section four and the conclusion in

section 5.

## 2. Literature

Empirical economic growth studies generally start with a variant of the following general specification:

$$\ln(Y_{it}) - \ln(Y_{it-\tau}) = \kappa + \beta \ln(Y_{it-\tau}) + W_{it-\tau}\delta + \eta_i + \zeta_t + \epsilon_{it} \quad (1)$$

Where  $Y_{it}$  is per capita GDP, in country  $i$  in the period  $t$ ,  $W_{it-\tau}$  is a file vector of economic growth determinants,  $\eta_i$  is the fixed component specific to each country,  $\zeta_t$  is a constant specific to each period, and  $\epsilon_{it}$  is the error term.

These types of equations used in cross-section estimates are referred to as the Barro regressions. Barro and Sala-i-Martin (1990) prove the existence of conditional convergence for the U.S.A. at a rate of around 2,5 % in the period 1840-1988. They also find evidence of a conditional convergence rate of around 2,0 % a year for a sample of 98 countries, in the period 1960-1985, calculated using least squares regressions. The problem with these results is that the non-observable individual effect is treated inadequately, since it should not be correlated to the other variables on the right-hand side, but in fact is. There is also a problem with the endogeneity of the regressors used.<sup>1</sup>

Caselli, Esquivel and Lefort (1996) introduced a new methodology in the study of economic growth and convergence, based on the estimation of dynamic data panels, which corrects endogeneity, omitted variable and consistency problems. By this, they induce a jump in the estimation of the convergence coefficient from 2 % to 10 % annually approximately, which implies that economies are always near their steady state. Of course, they use the same sample and period as Barro and

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<sup>1</sup>This will be explained in greater detail in the Methodology section

Sala-i-Martin (1990). This is also the case in the estimation of the simple Solow model and its augmented version, with variables such as initial income, average years of secondary schooling for men and women, life expectancy index logarithm, political assassinations, terms of trade among others. The Hausman test rejects the strict exogeneity of each set of variables for both models. As such, consistent parameters are obtained through the new technique.

Meanwhile, in his study on Chilean economic growth using the panel data methodology, Lefort (1997) gives an adequate treatment to the individual non-observable effects and to the endogeneity problems characteristic of these cross-section studies and finds that the increase in Chilean growth rates is mainly due to the direct effects of the economic reforms on the macro-economic variables, such as increase in investment, opening of the economy and greater efficiency of the financial system. Furthermore, he shows that the effect arising from the variation of the individual component not explained by other variables is insignificant in the Chilean case, and stands at around 1.6%, which would imply that we converge on a not very high per capita income level. The convergence rate estimated through least squares regressions is 2.3% and around 9% for GMM, which demonstrates the significant difference produced by the new methodology.

One of the last studies aimed at improving convergence results is Bond et al. (2001) which describes the general form of the approximation made by Caselli, Esquivel and Lefort (1996), stating the equation for regression as a model of dynamic data panels. It takes the first differences, removing the specific non-observable effect of each country and holding constant over time the instrument used as a variable in the right side, with lagged series of two periods or more in levels, under the assumption that the temporal variation of the errors in the

original equation in levels are not serially correlated. This permits consistent estimators to be obtained. The problem with this method is that when the time series are persistent and the number of observations is small, the first stage GMM estimator behaves poorly. Hence, using the estimator suggested by Arellano and Bover (1995) and Blundell and Bond (1998) they show that the results of the convergence coefficient are significantly lower than that found by Caselli, Esquivel and Lefort (1996), since it solves the bias problems of the first stage GMM estimator.

In summary, one may say that recent empirical studies have concentrated on dynamic panel techniques.<sup>2</sup>

### 3. Methodology

The previous section offered a brief overview of empirical growth studies. The common aspect of these studies is that they use the Solow-Swan model as the base, where savings rates, as well as population growth rates are exogenous. An aggregate production function is assumed with effective work and capital stock arguments. The growth rate behavior of countries around the steady state is given by the following expression:

$$\begin{aligned} \ln(Y_{i,t}) - \ln(Y_{i,t-\tau}) &= -(1 - \exp^{-\lambda\tau})(Y_{i,t-\tau}) + \\ &(1 - \exp^{-\lambda\tau})\frac{\alpha}{1 - \alpha}[\ln(s) - \ln(n + g + d)] + \eta_i + \epsilon_{i,t} \end{aligned} \quad (2)$$

where

$$\lambda = (n + g + d)(1 - \alpha) \quad (3)$$

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<sup>2</sup>It should be noted that there is a large body of literature on heterogeneous dynamic panels. In the present paper, we concentrate on homogenous panels that also satisfy the steady state/seasonality assumption.

with  $n$  is the population growth rate,  $g$  is the productivity-enhancing technological development rate,  $d$  is the fixed capital depreciation rate,  $\alpha$  is the fixed capital depreciation rate and  $s$  is the savings rate.  $\lambda$  is the convergence coefficient that measures the velocity at which countries converge to their steady state product level.

We know that one way of testing the Solow growth model, and to discover whether there is convergence or not, is by estimating a growth equation of the following type

$$\Delta y_{it} = \beta y_{it-r} + \gamma s_{it-r} + (\eta_i + \epsilon_{it}) \quad |\beta| < 1 \quad , \quad r > 1 \quad (4)$$

where  $\Delta y_{it}$  is per capita income growth, which is accounted for by the initial income level  $y_{it-r}$ , and by variables that serve to characterize the steady state of the economy. The assumption  $|\beta| < 1$ , demonstrates that we are working with steady state panels. Concordantly, the lag should be greater than one, given that if  $r = 1$ , the problem of testing convergence in the previous equation is reduced to proving the existence of unit root<sup>3</sup>. There are various forms for estimating (4), from the least squares regressions estimator to the Generalized Moment Method (henceforth referred to as GMM). Note that (4) may be estimated equivalently as

$$y_{it} = \tilde{\beta} y_{it-1} + \gamma s_{it-r} + (\eta_i + \epsilon_{it}) \quad (5)$$

If the least squares regression or within group estimator (henceforth referred to as WG) is used, the estimators are taken as inconsistent since they do not consider the problem of serial correlation that exists between the lagged dependent variable

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<sup>3</sup>It should be noted that Harris and Tzavalis (1999) developed a unit root test when the panel assumes a fixed T and  $N \rightarrow \infty$



$y_{it-\tau}$  and the error term  $\epsilon_{it}$ . The methodology for obtaining consistent estimators is GMM. A brief description of these methodologies that are used throughout this study is outlined below.

### 3.1. The Arellano and Bond method

For the sake of simplicity, we shall assume that the model in which we are interested is given by<sup>4</sup>

$$y_{it} = \alpha y_{it-1} + \eta_i + \epsilon_{it} \quad i = 1 \dots N \quad t = 2 \dots T \quad (6)$$

If we take the first difference of (6), to eliminate the term  $\eta$  the following is obtained:

$$\Delta y_{it} = \alpha \Delta y_{it-1} + \Delta \epsilon_{it} \quad i = 1 \dots N, \quad t = 3 \dots T \quad (7)$$

Arellano and Bond (1991) propose estimating the model above using GMM. The moment conditions considered are as follows:

$$E(\Delta \epsilon_{it} y_{it-s}) = 0 \quad i = 1 \dots N; \quad t = 3 \dots T; \quad s = 2 \dots t - 1 \quad (8)$$

where, based on the above, the availability of these is determined by  $m = \frac{(T-1)(T-2)}{2}$ . Stating the above conditions in matricial/matrix terms we have:

$$E(Z_i' \Delta \epsilon_i) = 0 \quad (9)$$

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<sup>4</sup>This is simply to facilitate the presentation of the various estimators; however, in the multi-variate case, the results hold, with the proviso that the nature of the regressors must be specified, in other words, if they are considered predetermined, exogenous or strictly exogenous, which is crucial in our case. Appendix A shows the treatment given when the variables may be exogenous, predetermined or endogenous.

where  $Z_i$  is a matrix of instruments of  $(T - 2) \times m$ , which is specified as:

$$Z_i = \begin{bmatrix} y_{i1} & 0 & 0 & \dots & \dots & 0 \\ 0 & y_{i1} & y_{i2} & \dots & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \dots & \cdot \\ 0 & 0 & 0 & y_{i1} & \dots & y_{iT-2} \end{bmatrix} \quad (10)$$

The GMM estimator based on the conditions defined by (8) and (9) minimizes the following criteria:

$$M = \left[ \frac{1}{N} \sum_{i=1}^N \Delta \epsilon'_i Z_i \right] W_N^{-1} \left[ \sum_{i=1}^N Z'_i \Delta \epsilon_i \right] \quad (11)$$

Where  $W_N$  is a matrix of weightings and its choice leads to two estimators that are asymptotically equivalent.<sup>5</sup> For the estimator of one stage, we use the following weighing matrix:

$$W_{N1} = \left[ \sum_{i=1}^N Z'_i H Z_i \right]^{-1} \quad (12)$$

where  $H$  is a matrix that contains two in the main diagonal and minus one in the first two subdiagonals and zeros in the all the other places; this is done to control the moving average term that is generated in the errors when differentiating. Equation (13) shows the estimator that is obtained when (11) is minimized with respect to  $\alpha$ :

$$\widehat{\alpha}_{1GMM} = [\Delta y'_{(-1)} Z W_{N1}^{-1} Z' \Delta y_{(-1)}]^{-1} [\Delta y'_{(-1)} Z W_{N1}^{-1} Z' \Delta y] \quad (13)$$

where  $\Delta y_{(-1)}$  is a vector of  $N(T - 2) \times 1$  given by  $\Delta y'_{(-1)} = (\Delta y'_{1(-1)}, \dots, \Delta y'_{N(-1)})'$ ,

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<sup>5</sup>They are usually known as the one and two stage estimators respectively.

in the same way that  $\Delta y' = (\Delta y'_1, \dots, \Delta y'_N)'$  which is also of the order of  $N(T - 2) \times 1$ , and finally  $Z' = (Z'_1, \dots, Z'_N)'$ , which is matrix of  $m \times N(T - 2)$ . The estimator  $\widehat{\alpha}_{1GMM}$  is consistent in the measure that  $N \rightarrow \infty$ , though it is not efficient. The efficient estimator, which we term  $\widehat{\alpha}_{2GMM}$  emerges from choosing the optimum weighing matrix, which has the following form:

$$W_{N2} = \left[ \frac{1}{N} \sum_{i=1}^N Z'_i \widehat{\Delta v}_i \widehat{\Delta v}_i' Z_i \right]^{-1} \quad (14)$$

where  $\widehat{\Delta v}_i$  are the estimated residuals based on a consistent estimator of  $\alpha$ , which is usually the  $\widehat{\alpha}_{1GMM}$  estimator. It should be noted that Arellano and Bond (1991) indicate that the  $\widehat{\alpha}_{2GMM}$  estimator presents a biased variance in finite samples and they therefore recommend inferences using the  $\widehat{\alpha}_{1GMM}$  estimator; notwithstanding the inference with  $\widehat{\alpha}_{2GMM}$  it may be done if the correction to the second stage estimator variance is carried out, as proposed by Windmeijer (2001).<sup>6</sup> It is thereby possible to use the efficient estimator when carrying out the inference.

### 3.2. The Blundell and Bond method

It is interesting to note that the estimators presented above may possess considerable bias if the coefficient associated to the lagged dependent variable is very near to one, in other words, if the series is highly persistent.<sup>7</sup> This occurs because the instruments become weak with highly persistent series; in other words, they no longer satisfy one of the conditions required from an instrument, which is that there must be a high correlation between it and the variable that will be instrumentalized. To solve this problem, Blundell and Bond derive an estimator known

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<sup>6</sup>This correction is implemented in the DPD of OX packet.

<sup>7</sup>It is important to indicate that if  $\eta_i$  is random and if its variance tends to infinite, then the estimators will also be biased.

as the system estimator, which combines the conditions of the estimator in first differences and the moment conditions of an estimator in levels <sup>8</sup> simultaneously. The moment conditions used are given by:

$$E(y_{it-s}\Delta\mu_{it}) = 0 \quad t = 2...T \quad s = 2....t - 1 \quad (15)$$

$$E(\Delta y_{it-1}\mu_{it}) = 0 \quad t = 3...T \quad (16)$$

with  $\mu_{it} = \eta_i + \epsilon_{it}$ . Using matrixes, we have:

$$E(Z'_{si}q_i) = 0$$

where  $Z_s$  is

$$Z_s = \begin{bmatrix} Z_{di} & 0 \\ 0 & Z_{li}^P \end{bmatrix} = \begin{bmatrix} Z_{di} & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta y_{i2} & 0 & \dots & \dots & 0 \\ 0 & 0 & \Delta y_{i3} & \dots & \dots & 0 \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \dots & 0 & \Delta y_{iT-1} \end{bmatrix} \quad (17)$$

where  $Z_{li}^P$  takes the elements of the diagonal/axis of the instrument matrix of the estimator in levels.

As in the earlier cases, the one and two stage estimator is obtained in the same manner as in the Arellano and Bond method.

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<sup>8</sup>For a review of this and other estimators see Benavente and Melo, who provide a review of the various methods used in dynamic panels

### 3.3. The Kiviet Correction

The previous estimators possess good asymptotic properties; however, in finite samples, their behavior may be very poor and display considerable bias. Kiviet (1995) attempts to solve this problem using the *LSDV* estimator, which is simply the least squares estimator with dummy variables.<sup>9</sup> While this *LSDV* estimator turns out to be inconsistent, Kiviet finds a way of eliminating its bias by usually using the first stage *GMM* estimators with consistent estimators of the self-regressive coefficient.<sup>10</sup> The superiority of this method is demonstrated by means of Montecarlo experiments. Kiviet (1995) shows that the bias may be calculated as<sup>11</sup>:

$$\begin{aligned}
E(\widehat{\beta}_{LSDV} - \beta) &= -\sigma_\epsilon^2(\overline{D})^{-1}\left(\frac{N}{T}(\iota_T' C \iota_T)[2q - \overline{W}' A \overline{W}(\overline{D})^{-1}q]\right. \\
&\quad + \text{tr}\{\overline{W}'(I_N \otimes A_T C A_T)\overline{W}(\overline{D})^{-1}\}q \\
&\quad + \overline{W}'(I_N \otimes A_T C A_T)\overline{W}(\overline{D})^{-1}q + \sigma_\epsilon^2 N q'(\overline{D})^{-1}q \\
&\quad \times \left[-\frac{N}{T}(\iota_T' C \iota_T)\text{tr}\{C' A_T C\} + 2\text{tr}\{C' A_T C A_T C\}\right]q \\
&\quad + O(N^{-1}T^{-3/2})
\end{aligned} \tag{18}$$

Where  $\text{tr}$  denotes the trace operator  $\overline{D} = \overline{W}P'A\overline{W} + \sigma_\epsilon^2 N \text{tr}C'A_T C q q'$ ,  $A_T = I_T - \frac{1}{T}\iota_T \iota_T'$ ,  $q = (1, 0, \cdot, \cdot, \cdot, 0)$ ,  $A\overline{W} = E(AW)$

The expression above has the disadvantage of being extremely complex. However, there is an alternative means of obtaining this correction, which was developed by Kiviet and Bun (2003). In order to apply this formula to the growth model in which we are interested, we shall assume the equation of interest to be:

$$y_{it} = \gamma y_{it-1} + \alpha x'_{jit} + \eta_i + \epsilon_{it}$$

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<sup>9</sup>It is simply the *WG* estimator.

<sup>10</sup>He uses the estimator in difference of Arellano and Bond and the system estimator, both in their first stage.

<sup>11</sup>For further details on the derivation of this expression, see Kiviet (1995).

where  $j = 1, 2$ , and represents the regressors to be considered in the estimate. However, we know that the parameter  $\gamma$ , may be seen as  $\gamma = 1 + \beta$ . through theoretical conditions of the Solow model. This is significant since the term of interest is  $\beta$ . This allows us to apply the Kiviet correction to a model such as:

$$\Delta y_{it} = \beta y_{it-1} + \alpha x'_{jit} + \eta_i + \epsilon_{it}$$

It is simple to show that the Kiviet methodology applies to the model above. Based on this, it is possible to define the bias component that should be removed, which is  $O(T^{-1})$ . This term may be stated as:<sup>12</sup>

$$C_1(T^{-1}) = \sigma_\epsilon^2 \text{tr}(\Pi) q_1$$

Where  $\Pi = AL\Gamma$ , with  $I_N \otimes \Gamma_T$  and  $\Gamma_T = (I_T - \beta L_T)^{-1}$ ,  $L = I_N \otimes L_T$ . The matrix  $L_T$ , xxxx has ones in the first subdiagonal and zeros in the other elements. Meanwhile,  $q_1$  is a vector of  $(k + 1) \times 1$  elements, which may be stated as  $q_1 = Qe_1$ , where  $Q = [\mathbb{E}(WW)]^{-1}$ , with  $Q = [Y_{(-1)}, X_1, X_2]$ , which gives a matrix of  $(k + 1) \times (k + 1)$ , where  $k$  represents the number of regressors apart from the lagged independent variable.<sup>13</sup> For vector  $e_1$  where  $e_1 = (1, 0, \dots, 0)$ , with  $(k + 1) \times 1$ . This formula is far easier to evaluate than that given by equation (18). The above correction gives us an estimated convergence rate with less bias. Appendix C of this paper, presents the results of a Montecarlo experiment we constructed, with the aim of displaying the superiority of this correction.

## 4. Results

The results of applying the methodologies indicated earlier are given in tables one, two and three of Appendix B. The data base used for carrying out the

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<sup>12</sup>The notation used here follows Kiviet and Bun's (2003) work closely.

<sup>13</sup> $k = 2$  in our experiment

estimates is the same as that used by Caselli, Esquivel and Lefort (1996).<sup>14</sup> The variables are measured in logarithms, where the savings rate is built as the ratio Investment-Product. Meanwhile, the effective population growth rate is built as the sum of the population growth rate plus the productivity enhancing technological development rate plus the physical capital depreciation rate, where the sum of the latter two,  $g + d$  is assumed to be 0.05. It should be noted that the variables are measured in five year periods, covering 97 countries, in the period 1960-1985. The advantage of having this base is that the results obtained are directly comparable to those reported by them. Table one shows the estimation of the Solow model. Columns one and two report the results of the traditional least squares/OLS and WG estimators. As was already mentioned in the methodology description, these estimators are inconsistent, however, they aid us in working with the GMM since the estimators obtained with this technique cannot go over the least squares estimator, and cannot go under the WG. When we proceed to estimate by GMM, the savings rate and population growth rate are treated as endogenous, and we therefore instrumentalize in line with this fact.<sup>15</sup> Considering this, we may observe that since column three uses the GMM estimator of differences, the parameter associated to the lagged per capita income is -0.47, which implies a convergence rate of around 12. As mentioned earlier, the Caselli et al (1996) work is replicated in column three. If we were to believe in these results, the implications for growth theory would be that countries complete their path to the steady state in a period of 10 years, which is surprisingly high compared to the convergence rate of other studies. Hence, columns five and six show the first and second stage system estimators respectively.<sup>16</sup> A result that immediately

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<sup>14</sup>The data for this base are obtained from Summer and Heston, and Barro and Lee.

<sup>15</sup>Appendix B indicates the way in which moment conditions are treated when we have exogenous, endogenous and predetermined variables.

<sup>16</sup>This exercise was developed by Bond et al (2001). However, his results are not comparable to those of Caselli et al (1996), since they obtain different results in quantitative terms, but

grabs ones attention is the sharp drop in the parameter associated to the lagged per capita income, falling from  $-0,47$  to  $-0,09$ . Obviously, the convergence rate also falls and reaches around 2%, which is in line with most studies on convergence, particularly those with cross-section analysis. On the other hand, column five shows us that the three parameters are statistically significant, which is corroborated by the second stage estimator. These results show that the conclusions reached by Caselli et al (1996), are based on an estimator with poor performance when faced with weak instruments, which leads to estimate bias.<sup>17</sup> However, as mentioned in the methodology section (and as demonstrated in Appendix C of the present paper), we can obtain a more reliable result in finite samples for both the systems estimator and the estimator in differences. To this end, we utilize the Kiviet correction.

Table two shows the results of applying this technique. However, the sample drops from 97 to 92 countries, in order to obtain a balanced panel. It may be observed that the first four columns do not greatly differ from that reported in table one. However, column five displays significant differences. In column five, the parameter associated to lagged per capita income is  $-0.15$ , which is more similar to the estimations of the Blundell and Bond estimator. The convergence obtained using this parameter is around 3%, which is once again in line with most of the empirical literature. This is interesting, since it gives robustness to the fact that the convergence rate is around 3 or 4% a year. Finally, we estimate an augmented Solow model, where the logarithm of the number of enrollments is included. The results are shown in table three, which once again confirm the fact

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similar results in qualitative terms, even though they use the same database. We started by replicating the results of Caselli et al (1996) and can thus compare our results to theirs, not only in qualitative terms but also in quantitative terms.

<sup>17</sup>By means of a Montecarlo experiment, its behavior, as well as that of other estimation methods for various parameters, is presented in Appendix C.



that when the correction is applied, the results do not differ greatly, confirming a convergence of around 3%.

## 5. Conclusions

New methodologies in dynamic panels for the estimation of growth equations are applied in the present paper. Hence, our point of departure was to replicate the results obtained by Caselli et al (1996) by applying the GMM systems estimator. We consequently found that the convergence rate of 12% dropped to around 3%, which indicates that there is no significant difference between the results of cross-section studies and our results. Furthermore, the results are robust, since when the Kiviet correction was applied, the convergence rate remained around 3%.

In summary, the final conclusion is that when estimating growth equations using GMM in dynamic panels, the results do not differ greatly from cross-section studies, and that the best way of estimating the relationship is by means of the Kiviet estimator.

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## Appendix A: Multivariate Analysis

The model we are interested in is of the following type:

$$y_{it} = \alpha y_{it-1} + \beta x_{it} + \mu_{it}, \quad t = 2, \dots, T$$

where  $\mu_{it} = \eta_i + v_{it}$  y  $x_{it}$  is a scalar. Let us assume that  $x_{it}$  is correlated to  $\eta_i$ . We also know that  $x_{it}$  may be correlated in three different ways to  $v_{it}$ , which will lead to various moment conditions. We shall firstly assume that  $x_{it}$  is strictly exogenous, and is expressed in the following manner:

$$E(x_{is}v_{it}) = 0 \quad \text{con} \quad s = 1, \dots, T, \quad t = 1, \dots, T$$

The second case may be that  $x_{it}$  is predetermined or weakly exogenous, where:

$$E(x_{is}v_{it}) = 0 \quad \text{con} \quad s = 1, \dots, t, \quad t = 1, \dots, T$$

and

$$E(x_{is}v_{it}) \neq 0 \quad \text{para} \quad s = t + 1, \dots, T$$

Finally,  $x_{it}$  may be determined endogenously, and so:

$$E(x_{is}v_{it}) = 0 \quad s = 1, \dots, t-1, \quad t = 1, \dots, T$$

and

$$E(x_{is}v_{it}) \neq 0 \quad s = t, \dots, T, \quad t = 1, \dots, T$$

With the above, we will have different moment conditions for each case, where the conditions given for the self-regressive part hold, but we will have additional conditions for each case of  $x_{it}$ . If  $x_{it}$  is strictly exogenous, the moment conditions are:

$$E(x_{its}\Delta v_{it}) = 0 \quad s = 1, \dots, T, \quad t = 3, \dots, T$$

which leads to the existence of  $T(T-2)$  additional moment conditions. Meanwhile, when  $x_{it}$  is predetermined, we have:

$$E(x_{is}\Delta v_{it}) = 0 \quad s = 1, \dots, t-1 \quad t = 3, \dots, T$$

where the additional moment conditions are  $0,5(T - 2)(t - 1)$ . Finally, for the case of endogenous  $x_{it}$  we have:

$$E(x_{is}\Delta v_{it}) = 0 \quad s = 2, \dots, t - 1 \quad t = 3, \dots, T$$

the available conditions are  $0,5(T - 2)(T - 1)$ . Thus, the systems estimator is obtained by combining the conditions in first differences and in levels. To demonstrate this, let us assume that  $x_{it}$  is endogenous, and that consequently the conditions in first differences are:

$$E(y_{it-s}\Delta v_{it}) = 0$$

and

$$E(y_{it-1}(\eta_i + v_{it}))$$

with  $t = 3 \dots T$  and  $s = 2, \dots, t - 1$

For  $x_{it}$  we have:

$$E(x_{it-s}\Delta v_{it}) = 0$$

and

$$E(\Delta x_{it-1}(\eta_i + v_{it})) = 0$$

for  $t = 3 \dots T$  and  $s = 2, \dots, t - 1$

## Appendix B: Estimate Results

**Tabla 1: Estimaciones del modelo de Solow**

	WG	OLS	DIF1	DIF2	SYS1	SYS2
Ingreso Per Capita(-1)	-0.32 (0.06)***	-0.03 (0.01)***	-0.47 (0.14)***	-0.31 (0.20)	-0.09 (0.04)**	-0.11 (0.03)***
Tasa de Ahorro	0.13 (0.04)***	0.09 (0.02)***	0.04 (0.07)	(0.11) 0.08	0.17 (0.04)***	0.19 (0.04)***
Tasa de Crecimiento de la población	-0.10 (0.15)	-0.12 (0.056)**	-0.20 (0.33)	-0.37 (0.32)	(-0.50)** (0.25)**	-0.53 (0.21)***
Lamda Implicado	0.077	0.006	0.1269	0.074	0.018	0.02
Wald			0.002**	0.074	0.0 **	0.0 **
Sargan			0.104	0.130	0.0**	0.76
AR(1)			0.005**	0.004**	0.0 **	0.0 **
AR(2)			0.655	0.796	0.81	0.78
N	97	97	97	97	97	97
Observaciones	477	477	380	380	477	477

\*\*\* Significativo al uno por ciento

\*\*Significativo al cinco por ciento

\*Significativo al diez por ciento

Entre parentesis errores estandar



Tabla 2: Estimaciones del modelo de Solow usando la Corrección de Kiviet

	WG	OLS	DIF1	DIF2	SYS1	SYS2	KIV1ET SYS1
Ingreso Per Capita(-1)	-0.31 (0.06)***	-0.03 (0.01)**	-0.46 (0.13)***	-0.35 (0.17)**	-0.10 (0.04)**	-0.10 (0.03)***	-0.15 (0.06)***
Tasa de Ahorro	0.13 (0.04)**	0.09 (0.02)***	0.05 (0.07)	0.10 (0.08)	0.17 (0.05)***	0.18 (0.04)***	0.11 (0.04)***
Tasa de Crecimiento de la población	-0.11 (0.16)	-0.12 (0.06)**	-0.23 (0.33)	-0.28 (0.28)	-0.49 (0.26)*	-0.40 (0.21)*	-0.07 (0.15)*
Lamda Implicado	0.074	0.006	0.123	0.086	0.021	0.021	0.032
Wald			0.001***	0.024**	0.0***	0.0***	
Sargan			0.121	0.157	0.03**	0.652	
AR(1)			0.004**	0.006**	0.0***	0.0***	
AR(2)			0.714	0.810	0.872	0.889	
N	92	92	92	92	92	92	92
Observaciones	460	460	368	368	460	460	460

\*\*\* Significativo al uno por ciento

\*\*Significativo al cinco por ciento

\*Significativo al diez por ciento

Entre parentesis errores estandar

Tabla 3: Estimaciones del modelo de Solow aumentado usando la Corrección de Kiviet

	WG	OLS	DIF1	DIF2	SYS1	SYS2	KIVETSYS1
Ingreso Per Capita(-1)	-0.32 (0.06)***	-0.05 (0.02)***	-0.27 (0.12)**	-0.30 (0.14)**	-0.06 (0.07)	-0.08 (0.03)***	-0.15 (0.07)***
Tasa de Ahorro	0.14 (0.04)***	0.08 (0.02)***	0.11 (0.06)*	0.18 (0.06)***	0.18 (0.04)***	0.17 (0.04)***	0.11 (0.04)***
Tasa de Crecimiento de la población	-0.03 (0.15)	-0.09 (0.05)	0.45 (0.48)	0.25 (0.53)	-0.38 (0.26)	-0.41 (0.20)**	0.003 (0.20)
Matricula	-0.05 (0.03)*	0.03 (0.01)***	-0.24 (0.10)***	-0.19 (0.12)	-0.04 (0.05)	-0.03 (0.05)	-0.042 (0.04)
Lamda implicado	0.077	0.010	0.062	0.071	0.012	0.016	0.033
Wald			0.001***	0.015**	0.00***	0.00**	
Sargan			0.687	0.66	0.014**	0.451	
AR(1)			0.005***	0.004**	0.00***	0.00**	
AR(2)			0.977	0.943	0.95	0.931	
N	92	92	92	92	92	92	92
Observaciones	460	460	368	368	460	460	460

\*\*\* Significativo al uno por ciento

\*\*Significativo al cinco por ciento

\*Significativo al diez por ciento

Entre parentesis errores estandar

## Appendix C: Montecarlo Simulation

The Montecarlo simulation that allows the superiority of the Kiviet correction to be demonstrated is described in this appendix. The experiment design combines elements from the works of Kiviet (1995), Kiviet and Bun (2003), as well as from Arellano and Bond (1991). The aim of this is to demonstrate that the convergence rate is estimated with less bias, through the corrected within group estimator proposed by Kiviet. This correction is fundamental for the results shown in Appendix B of this paper. As was mentioned in the methodology section, the Kiviet correction is applied to the within group estimator, which, as is well known, is inconsistent for a fixed  $T$ . This is expected to be fulfilled given the traits of the database that we are using, so the estimates obtained from the within group methodology are inconsistent. Given this, it is clear that the elimination of this bias component that depends on  $T$  would allow us to obtain a better and more precise form of estimating the convergence rate. This is exactly what Kiviet (1995) and Kiviet and Bun (2003) implement, which is simply eliminating the bias component and using the within group estimator in panels, with the characteristics that we are working with. In order to demonstrate the superiority of this correction, a Montecarlo experiment is designed, where the various estimation methods used in this paper are compared. The experiment and the parameters used are described as follows.

## The model

The model we are interested in is given by the following dynamic specification:

$$y_{it} = \gamma y_{it-1} + \beta x'_{jit} \eta_i + \epsilon_{it}$$

Where  $i = 1, \dots, N$ ,  $t = 1, 2, \dots, T$ ,  $j = 1, 2$ , which reflects the fact that we have two regressors in the equation for estimation. As indicated earlier, there are several methods for estimating this model. As already stated, the experiment that we carry out here closely follows Arellano and Bond (1991), Kiviet (1995), and Kiviet and Bun (2003). The aspects to be considered are:

1.  $y_{i0} = 0$ , is not fixed, but is instead left random. In order to prevent this assumption from having a significant effect on the results, the first 50 observations of each unit are eliminated. This gives us the necessary flexibility in the issue of the initial observation for each process of the corresponding unit, so that the results do not depend on this borderline condition.
2. The term  $\epsilon_{it}$  is generated with  $\sigma_{\epsilon}^2 = 1$ , and  $\sigma_{\eta}^2 = 2^{18}$
3. In contrast to Kiviet (1995), Kiviet and Bun (2003), and Arellano and Bond (1991), our design has two regressors associated to the lagged dependant variable. This allows us to know the properties of the estimators in the face of changes in the structure of the regressors. Thus, we assume that  $\sigma_{\xi_{x_1}}^2 = \sigma_{\xi_{x_2}}^2 = 1$ . The values of  $\beta_1$  and  $\beta_2$  are allowed to differ from each other in the experiment. We assume that  $\beta_1 = 0,9$  and  $\beta_2 = 0,59$  for the savings rate and population growth rates respectively. These values arise from the database that we are using. The objective of this is to be able to feed the model with the structure of the real data, and to know how the various estimators behave under these conditions.

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<sup>18</sup>Realizamos diversas parametrizaciones y los resultados no varían mayormente.

4. Finally, we fixed  $N = 92$  and  $T = 5$ . These parameters come from the database that generates the results displayed in Appendix B.

In summary, it may be said that the design of the experiment is sufficiently rich in order to obtain the necessary conclusions regarding the properties of the estimators in which we are interested. This is because both theoretical and real data elements are combined.

### Results of the Simulated Model

Tables A.3.1, A.3.2, A.3.3 and A.3.4, display the results of the Montecarlo experiment described above<sup>19</sup> The exercise includes various values of the  $\gamma$ , coefficient, as well as estimations for various methodologies. The columns of the tables show the results for the estimates using least squares, WithinGroup (*LSDV*), Kiviet estimator, Arellano and Bond estimator (*GMM*), and the Blundell and Bond systems estimator (*GMM – SYS*). In order to analyze the robustness of the experiment in which we are interested, we replicated the Kiviet (1995) experiment, but with two regressors instead. Table A.3.1 shows the results in terms of bias, where it is clear that the Kiviet estimator shows the lowest average bias for the various parameters. It should be highlighted that the GMM-SYS, always overestimates the true value of the parameter and its behavior is very similar to the OLS. On the other hand, when we use the Mean Quadratic Error criterion, it can again be seen in Table A.3.2 that the Kiviet estimator is the best estimator for all parameters. With this in mind, the results obtained from the simulation in which we are interested may be analyzed.

In Table A.3.3, it can be clearly seen that regarding bias, the Kiviet estimator is superior to all other estimators. The Kiviet model particularly performs better than the Blundell-Bond estimator systems and OLS for values of  $\gamma = 0,95$ , which is a critical value for this type of model. This result is of vital interest for

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<sup>19</sup>The number of repetitions is 100 simulations.

the estimations of the growth models shown in Appendix B of this paper. The behavior of the Kiviet and GMM-SYS estimators are not significantly different for the coefficients  $\beta_1$  y  $\beta_2$ , Thus, these underline the fact that the Kiviet method offers the lowest bias when estimating the convergence rate in the Solow model. This reaffirms the results shown in Appendix B.

Nevertheless, it is also possible to carry out the comparison in terms of the Mean Quadratic Error. Once again, the results here confirm the fact that the Kiviet estimator is superior to the GMM-SYS estimator, since the results show that it has the lowest Mean Quadratic Error for the various values of  $\gamma$ , The Kiviet is also superior in the case of the  $\beta_1$  and  $\beta_2$ , coefficients, the behavior of the ECM.

Certain observations should be made concerning the results reported in Tables A.C.3 and A.C.4. Firstly, the poor performance of the Within Group and GMM estimators should be mentioned. This is a standard result in the literature and should not surprise, but rather delivers robustness to the main results arising from this experiment.

The results provided herein allow us to be confident of the supremacy of the Kiviet estimator of growth models. It provides robustness to the convergence rates reported in Appendix B of the present paper. Lastly, it demonstrates the need to evaluate this type of estimate through Montecarlo experiments, attempting to capture the elements of the real data available to the researcher.

**Table A.C.1: Comparison of the mean bias for various values of  $\gamma$ ,  
using the Kiviet (1995) experiment**

	$\gamma$	OLS	LSDV	KIVIET	GMM	GMM-SYS
Bias $\gamma$	0.70	0.272	-0.365	0.227	-0.390	0.290
	0.75	0.225	-0.395	0.197	-0.460	0.242
	0.80	0.178	-0.427	0.167	-0.546	0.193
Bias $\beta_1$	0.70	-0.200	0.052	-0.029	0.043	-0.145
	0.75	-0.158	0.046	-0.019	0.024	-0.110
	0.80	-0.118	0.042	-0.011	0.030	-0.084
Bias $\beta_2$	0.70	-0.080	-0.010	-0.008	0.003	-0.134
	0.75	-0.060	-0.014	-0.008	-0.015	-0.103
	0.80	-0.045	-0.019	-0.017	-0.019	-0.097

**Table A.C.2: Comparison of the Mean Quadratic Error for various values of  $\gamma$ , using the Kiviet (1995) experiment.**

	$\gamma$	OLS	LSDV	KIVIET	GMM	GMM-SYS
E.C.M $\gamma$	0.70	0.273	0.368	0.237	0.439	0.291
	0.75	0.225	0.397	0.207	0.508	0.242
	0.80	0.178	0.430	0.178	0.592	0.193
E.C.M $\beta_1$	0.70	0.205	0.123	0.126	0.367	0.221
	0.75	0.165	0.139	0.144	0.426	0.215
	0.80	0.146	0.230	0.242	0.741	0.321
E.C.M $\beta_2$	0.70	0.115	0.119	0.130	0.283	0.328
	0.75	0.110	0.138	0.150	0.325	0.356
	0.80	0.160	0.235	0.255	0.547	0.570



**Table A.C.3: Comparison of the mean bias for various values of  $\gamma$ ,  
using various estimation methods**

	$\gamma$	OLS	LSDV	KIVJET	GMM	GMM-SYS
Bias $\gamma$	0.70	0.296	-0.291	0.213	-0.304	0.299
	0.75	0.247	-0.296	0.184	-0.333	0.249
	0.80	0.198	-0.299	0.154	-0.357	0.200
	0.85	0.149	-0.300	0.123	-0.366	0.150
	0.90	0.100	-0.290	0.089	-0.294	0.100
	0.95	0.054	-0.045	0.007	-0.007	0.054
	0.97	0.038	-0.007	0.001	-0.001	0.038
	Bias $\beta_1$	0.70	-0.079	0.013	-0.008	0.005
0.75		-0.075	0.012	-0.006	-0.005	-0.041
0.80		-0.069	0.010	-0.004	-0.020	-0.033
0.85		-0.062	0.008	-0.002	-0.042	-0.024
0.90		-0.051	0.003	0.000	-0.061	-0.007
0.95		-0.036	-0.001	0.001	-0.002	0.016
0.97		-0.028	-0.000	0.000	0.000	0.019
Bias $\beta_2$		0.70	0.049	0.000	-0.004	0.001
	0.75	0.043	0.003	-0.005	0.011	0.031
	0.80	0.037	0.006	-0.006	0.024	0.019
	0.85	0.029	0.009	-0.007	0.038	0.009
	0.90	0.020	0.012	-0.007	0.042	-0.003
	0.95	0.011	0.000	-0.003	-0.001	-0.010
	0.97	0.008	-0.002	-0.002	-0.002	-0.004

Table A.C.4 Comparison of the Mean Quadratic Error for various values of  $\gamma$ , using various estimation methods

	$\gamma$	OLS	LSDV	KIVIET	GMM	GMM-SYS
E.C.M $\gamma$	0.70	0.296	0.294	0.223	0.345	0.299
	0.75	0.247	0.299	0.194	0.375	0.249
	0.80	0.198	0.302	0.165	0.400	0.200
	0.85	0.149	0.303	0.134	0.410	0.150
	0.90	0.100	0.293	0.101	0.346	0.100
	0.95	0.054	0.048	0.019	0.021	0.054
	0.97	0.038	0.009	0.006	0.007	0.038
E.C.M $\beta_1$	0.70	0.080	0.029	0.029	0.087	0.066
	0.75	0.076	0.029	0.028	0.088	0.060
	0.80	0.071	0.028	0.028	0.091	0.053
	0.85	0.063	0.028	0.027	0.098	0.046
	0.90	0.053	0.027	0.027	0.116	0.036
	0.95	0.038	0.027	0.026	0.088	0.045
	0.97	0.031	0.026	0.026	0.088	0.056
E.C.M $\beta_2$	0.70	0.054	0.027	0.032	0.072	0.085
	0.75	0.048	0.027	0.031	0.075	0.078
	0.80	0.042	0.027	0.031	0.082	0.071
	0.85	0.035	0.028	0.031	0.090	0.066
	0.90	0.029	0.030	0.030	0.097	0.063
	0.95	0.022	0.029	0.028	0.069	0.059
	0.97	0.021	0.028	0.028	0.068	0.064