EXISTENCE AND UNIQUENESS OF THE COMPETITIVE EQUILIBRIUM FOR INFINITE DIMENSIONAL ECONOMIES

Elvio Accinelli*

ABSTRACT

In this paper we prove the existence of an equilibrium theorem and we obtain a condition for uniqueness of equilibrium in infinite dimensional economies using the excess utility function.

SINTESIS

En este trabajo se demuestra la existencia de un teorema de equilibrio y se obtiene una condición de unicidad de equilibrio en economías de infinitas dimensiones recurriendo a la función de exceso de utilidad

IMERL, Facultad de Ingeniería, Montevideo, Uruguay. **I wish to thank Aloisio Araujo, Paulo K. Monteiro, Flavio Menezes, Daniel Vaz and Ricardo Marchesini for their useful comments.**

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1. INTRODUCTION

In this paper without assuming the existence of the demand function, from the excess utility function, we prove the existence of an equilibrium theorem, and we obtain a condition to uniqueness of equilibrium.

In the first section we characterize the model, and we introduce some standard definitions from the general equilibrium theory.

In the second section we introduce the excess utility function and we show some of its properties.

In the third section, from the excess utility function, we prove that there exist a bijective relation between the equilibrium allocations set and the set of zeroes in the excess utility fuction.

In the fourth part, from the excess utility function we obtain a binary relation in the social weights space, and prove that the equilibrium set is not empty. Our main tool is the Knaster, Kuratowski, Masurkiewicz lemma.

In the next section, from the excess utility function, we define the weak axiom of the revealed preference. So defined, this axiom, is only formally similar to the classical one. It has the same mathematical properties as those of the classical axiom of revealed preference, but it does not have the same economic interpretation. We prove that if the excess utility function has this property then uniqueness of equilibrium follows, that is, there exists only one zero for this function.

Finally, examples of economies with the weak axiom of revealed preference in the excess utility function are given.

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2. THE MODEL

Let us consider a pure exchange with n agents and l goods at each state of the world. The set of states is a measure space: (Ω, A, ν).

We assume that each agent has the same consumption space, $M = \Pi_{j=1}^{l} M_j$ where M_j is the space of all positive measurable functions defined on (Ω, A, ν).

Let $R^l_{++} = \{ x \in R^l$ with all components positive$\}$.

Following Mas-Collel (1989), we consider the space Λ of the C^2 utility functions on R^l_{++}, strictly monotone, differentiably strictly concave and proper.

Definition 1. *A C^2 utility function u is diferentiably strictly convex, if it is strictly convex and every point is regular; that is, the Gaussian curvature, C_x, of each level surface of u, is a non null function in each x.*

For $x, y \in R^l$ we will write $x > y$ if $x_i \geq y_i$ $i = 1 \ldots l$ and $x \neq y$.

Definition 2. *A utility function is strictly monotone if $x > y \Rightarrow u(x) > u(y)$.*

Definition 3. *We say that $u \in C^2$ is proper if the limit of $|u'(x)|$ is infinite, when x approaches the boundary of R^l_{++}, i.e. the set $B = \{ x: x_i = 0$ for some $i = 1, \ldots, n \}$.*

We will consider the space U of all measurable functions $U : \Omega \times R^l_{++} \to R$, such that $U(s, .) \in \Lambda$ for each $s \in \Omega$.

We introduce the uniform convergence in this space: $U_n \to U$ if $\|U_n - U\|_K \to 0$ for any compact $K \subset R^l_{++}$, where $\|U_n - U\|_K =$

$$\operatorname*{ess\,sup}_{s\in\Omega} \max_{z\in K} \{|U_n(s,z) - U(s,z)| + |\partial U_n(s,z) - \partial U(s,z)| + |\partial^2 U_n(s,z) - \partial^2 U(s,z)|\}$$

Each agent is characterized by his utility function u_i and by his endowment $w_i \in M$. From now on one we will with economies with the following characteristics:

a) The utility functions $u_i : M \to R$ are separable. This means that they can be represented by

$$u_i(x) = \int_\Omega U_i(s, x(s))\, dv(s) \quad i = 1,\dots, n \tag{1}$$

where $U_i : \Omega \times R^l_{++} \to R$ and $U_i(s, .)$ is the utility function of each agent at every state $s \in \Omega$.

b) The utility functions $U_i(s, .)$ belongs to a fixed compact subset of Λ, for each $s \in \Omega$ and $U_i \in U$.

c) The agents' endowments, $w_i \in M$ are bounded above and bounded away from zero in any component, i.e., there exists h and H with $h < w_{i_j}(s) < H$ for each $j = 1\dots l$, and $s \in \Omega$.

The following definitions are standard.

Definition 4. *An allocation of commodities is a list $x = (x_1, \dots, x_n)$ where $x: \Omega \to R^{ln}$ and $\sum_{k=1}^n x_k(s) \leq \sum_{k=1}^n w_k(s)$.*

Definition 5. *A commodity price system is a mesurable function $p : \Omega \to R^l_{++}$, and for any $z \in R^l$ we denote by $\langle p, z\rangle$ the real number $\int_\Omega p(s)\, z(s)\, dv(s)$. (We are not using any specific symbol for the Euclidean inner product in R^l.)*

Definition 6. *The pair (p, x) is an equilibrium if:*

i) p is a commodity price system and x is and al location,

ii) $\langle p, x_i\rangle \leq \langle p, w_i\rangle < \infty \ \forall\, i \in \{1,\dots, n\}$

iii) if $\langle p, z\rangle \leq \langle p, w_i\rangle$ with $z: \Omega \to R^l_{++}$, then

$$\int_\Omega U_i(s, x_i(s))\, dv(s) \geq \int_\Omega U_i(s, z(s))\, dv(s) \ \forall\, i \in \{1,\dots n\}.$$

2. THE EXCESS UTILITY FUNCTION

In order to obtain our results we introduce the excess utility function.

We begin by writing the following well known proposition:

Proposition 1. *For each λ in the (n - 1) dimensional open simplex, $\Delta^{n-1}=\{\lambda \in R^n_{++}; \sum \lambda_i = 1\}$, there exists $\bar{x}(\lambda)=\{\bar{x}_1(\lambda),\ldots,\bar{x}_n(\lambda)\}\in R^{ln}_{++}$ that solves the following problem:*

$$\max_{x\in R^{l}{}_{n}} \textstyle\sum_i \lambda_i\, U_i\,(x_i) \tag{2}$$
$$\textit{subject to } \textstyle\sum_i x_i \le \sum_i w_i \textit{ and } x_i \ge 0 \cdot$$

If U_i depend also on $s \in \Omega$, and $U_i(s, .) \in \Lambda$ for each $s \in \Omega$, and $\lambda \in \Delta^{n-1}$, there exists $\bar{x}\,(s, \lambda) = \bar{x}_1\,(s, \lambda),\ldots, \bar{x}_n\,(s, \lambda)$ that solves the following problem:

$$\max_{x(s)\in R^{l}{}_{n}} \textstyle\sum_i \lambda_i\, U_i\,(s, x_i(s)) \tag{3}$$
$$\textit{subject to } \textstyle\sum_i x_i\,(s) \le \sum_i w_i\,(s) \textit{ and } x_i(s) \ge 0.$$

If $\gamma^j\,(s, \lambda)$ are the Lagrange multipliers of the problem (3), $j \in \{1,\ldots l\}$, then from the first order conditions we have that

$$\lambda_i \frac{\partial U_i\,(s, x\,(s,\lambda))}{\partial x^j} = \gamma^j\,(s, \lambda) \text{ with } i \in \{1,\ldots, n\} \text{ and } j \in \{1,\ldots l\}$$

Then the following identities hold

$$\lambda_i\, \partial U_i\,(s, x\,(s, \lambda)) = \gamma(s, \lambda)\ \forall\ i = 1,\ldots, n; \text{ and } \forall\ s \in \Omega \tag{4}$$

Remark 1 *From the Inada condition of "infinite marginal utility" at zero (Definition 3), the solution of (3) must be strictly positive almost everywhere. Since U(s, .) is a monotone function, we can deduce that $\sum_{i=1}^n \bar{x}_i(s) = \sum_{i=1}^n w_i(s)$.*

Let us now define the excess utility function.

Definition 7. *Let $x_i\,(s, \lambda)$; $i \in \{1,\ldots, n\}$ be a solution of (3).*

We say that $e : \Delta^{n-1} \to R^n\ e\,(\lambda) = (e_1\,(\lambda),\ldots, e_n\,(\lambda))$, *with*

$$e_i\,(\lambda) = \frac{1}{\lambda_i} \int_\Omega \gamma(s, \lambda)\,[x_i\,(s, \lambda) - w_i\,(s)]\; d\nu\,(s),\ i = 1,\ldots n \tag{5}$$

is the excess utility function.

Lemma 1. *The excess utility function is bounded for above, that is, there exists $k \in R$ such that $e(\lambda) \leq k1$ where 1 is a vector with all its components equal to 1.*

Proof: To prove this property, note that by definition we can write

$$e_i(\lambda) = \int_\Omega \partial U_i(s, x_i(\lambda))[x_i(s, \lambda) - w_i(s)]\, dv(s).$$

From the concavity of U_i it follows that:

$$U_i(s, x(s, \lambda)) - U_i(s, w(s)) \geq \partial U_i(s, x(s, \lambda))(x_i(s, \lambda) - w(s)).$$

Therefore,

$$e_i(\lambda) \leq \int_\Omega u_i(S, X_i(S, \lambda)) - u_i(W_i(S))\, DV(S) \leq \int_\Omega U_i\left(\sum_{j=1}^{n} w_j(s)\right) dv(s), \forall \lambda$$

If we let

$$k_i = \int_\Omega U_i\left(\sum_{i=1}^{n} w_i(s)\right) dv(s) \text{ and } k = \sup_{1 \leq i \leq n} k_i$$

the property follows.

Remark 2. *Since the solution of (3) is homogeneous of degree zero: i.e., $\bar{x}(s, \lambda) = \bar{x}(s, \alpha\lambda)$ for any $\alpha > 0$, then we can consider e_i defined all over the space R^n_{++} by $e_i(\alpha, \lambda) = e_i(\lambda)$ for all $\lambda \in \Delta^{n-1}_{++}$.*

3. EQUILIBRIUM AND EXCESS UTILITY FUNCTION

Let us now consider the following problem:

$$\max_{x \in M} \sum_i \bar{\lambda}_i \int_\Omega U_i(s, x_i(s))\, dv(s) \tag{6}$$
$$\text{subject to } \sum_i x_i(s) \leq \sum_i w_i(s) \text{ and } x_i(s) \geq 0$$

It is a well known proposition that an allocation $\bar{x}$ is Pareto optimal, if and only if we can choose a $\bar{\lambda}$ such that $\bar{x}$ solves the above problem, with $\lambda = \bar{\lambda}$. Moreover, since a consumer with zero social weight receives nothing of value as a solution of this problem, we have that if $\bar{x}$ is a strictly positive allocation, that is $\{\bar{x} \in R^l_{++}\}$, all consumption has a positive social weight. See for instance

Kehoe (1991). Reciprocally, if $\bar{\lambda}$ is in the interior of the simplex, then from remark (1) the solution $x(.\,,\lambda)$ of (6) is a strictly positive Pareto optimal allocation. (This is guaranteed also by the following boundary condition on preferences: $\{v(s) \in R^{l}_{++} : v(s) \succeq_i w_i(s)\}$ for all i and $w_i(s)$ strictly positive and is closed for a.e.s.).

From the first theorem of welfare, we have that every equilibrium allocation is Pareto optimal.

Let $(\bar{x})$ be an equilibrium allocation, then there exists a $\bar{\lambda}$ such that $\bar{x} = \{\bar{x}_1, \dots, \bar{x}_n\} : \Omega \to R^n$, is a solution for the problem at the beginning of this section.

In the conditions of our model, the first order conditions for this problem are the same that for (3). Then if a pair $(\bar{p}, \bar{x})$ is a price-allocation equilibrium, there exists a $\bar{\lambda}$ such that $\bar{x}(s) = \bar{x}(s, \bar{\lambda})$ solves (6), and $\bar{p}(s) = \gamma(s, \bar{\lambda})$, solves (4) for a. e. s.

Moreover, we have the following proposition:

Proposition 2. *A pair $(\bar{p}, \bar{x})$ is an equilibrium, if and if there exists $\bar{\lambda} \in \Delta^{n-1}$ such that $\bar{x}(s) = \bar{x}(s, \bar{\lambda})$ solves (6), and $\bar{p}(s) = \gamma(s, \bar{\lambda})$ solves (4) for a.e.s. and $e(\bar{\lambda}) = 0$.*

Proof: Suppose that $\bar{x}(.,\bar{\lambda})$ solves (6) and $\gamma(s, \bar{\lambda})$ solves (4). If for $\bar{\lambda} \in \Delta^{n-1}$, we have that $e(\bar{\lambda}) = 0$ then the pair $(\bar{p}, \bar{x})$, with $\bar{p} = \gamma(.\,, \bar{\lambda})$ and $\bar{x} = x(., \bar{\lambda})$ is an equilibrium.

Reciprocally, if $(\bar{p}, \bar{x})$ is an equilibrium, then it is straightforward by definition that $e(\lambda) = 0$. From de first welfare theorem, there exists $\bar{\lambda} \in \Delta^{n-1}$, such that $\bar{x}$ is a solution for (6). Since p is an equilibrium price, it is a support for $\bar{x}$, i.e., if for some x we have that $u_i(x) \geq u_i(\bar{x})$, $i = \{1, \dots n\}$, strictly for some i then $\langle \bar{p}, x_i \rangle > \langle \bar{p}, w_i \rangle$ and from the first order conditions we have that: $\bar{p}(s) = \gamma(s)$. The proposition is proved.

Let $\Delta^{n-1}_{++} = \{\lambda \in \Delta^{n-1} : \lambda_i > 0 \ \forall\, i = 1, \dots, n\}$.

We will now state the definition of the equilibrium set.

Definition 8. *We will say that* λ *is an equilibrium for the economy if* $\lambda \in E$, *where* $E = \{ \lambda \in \Delta_{++}^{n-1} : e(\lambda) = 0\}$. *The set E will be called, the equilibrium set of the economy.*

4. A BINARY RELATION IN THE SOCIAL WEIGHTS SPACE

Let $e: R^n \to R^n$ be an excess utility function.

Let us define $\succ$ in $\Delta_\epsilon^{n-1} = \{\lambda \in R_+^n : \sum_{i=1}^n \lambda^i = 1; \lambda_i \geq \epsilon\}$, a subset of the social weights space.

Definition 9. *We define* $\succ$ as:

$(\lambda_1, \lambda_2) \in \succ$ *iff* $\lambda_1 e(\lambda_2) < 0$.

We will write $\lambda_1 \succ \lambda_2$.

Properties of the Binary Relation $\succ$

$\succ$ *is irreflexible, convex, and upper semicontinuous.*

- *irreflexible* $\lambda \not\succ \lambda$ because $\lambda . e(\lambda) = 0$.
- *convex if* $\lambda^1 \succ \lambda$ and $\lambda^2 \succ \lambda$, then $\alpha\ \lambda^1 + \beta\ \lambda^2 \succ \lambda$ with $\alpha + \beta = 1$
- *upper semicontinous,* $A = \{\alpha \in \Delta_\epsilon^{n-1};\ \lambda \succ \alpha\}$ is open

Proof:

$A = \{\alpha \in \Delta_\epsilon^{n-1};\ \lambda . e(\alpha) < 0\}$

by the continuity of $\lambda . e(.)$, there exists an open neighbourhood V_α of α, such that $\lambda . e(V_\alpha) < 0$. Then A is open.

5. EXISTENCE OF EQUILIBRIUM

Definition 10. *We say that* γ *is a maximal element of* $\succ$ *if there does not exist a* λ *such that a* $\lambda \succ \gamma$.

Lemma 2. *The set of maximal elements in $\bar{\Delta}_\epsilon^{n-1}$ is non-empty.*

Proof: Note that

$$F(\lambda) = \bar{\Delta}_\epsilon^{n-1} - \{\alpha \in \bar{\Delta}_\epsilon^{n-1} \text{ such that } \lambda \succ \alpha\} = \{\alpha \in \bar{\Delta}_\epsilon^{n-1} \text{ such that } \lambda \,.e(\alpha) \geq 0\}$$

is a compact set.

We can also see that the convex hull of $\{\lambda_1,\ldots,\lambda_k\}$ is contained in $U_{i=1}^k F(\lambda_i)$ for all finite subsets $\lambda_1,\ldots \lambda_k \in \bar{\Delta}_\epsilon^{n-1}$. To this end, let $\lambda_1,\ldots, \lambda_k \in \bar{\Delta}_\epsilon^{n-1}$. If $\gamma = \sum_{i=1}^k a_i\lambda_i$ is a convex combination and γ is not in $U_{i=1}^k F(\lambda_i)$, then $\lambda_i \succ \gamma$ for every $i = 1,\ldots, n$, and so, since $\succ$ is convex value, we must have $\gamma \succ \gamma$. This is not possible because $\succ$ is irreflexible.

Then from the Fann-Theorem[1], it follows that $\bigcap_{\lambda \in \bar{\Delta}_\epsilon^{n-1}} F(\lambda) \neq \emptyset$. It is easy to see that the set of maximal elements in Δ_ϵ^{n-1} is equal to $\bigcap_{\lambda \in \bar{\Delta}_\epsilon^{n-1}} F(\lambda)$.

Then the theorem follows.

Theorem 1. *Let ϵ be an economy with infinite dimensional consumption space, with differentiable strictly convex C^a and separable utilities. Then ϵ has a non-empty, compact set of equilibrium.*

Proof: From lemma 2 we know that there exists γ_{ϵ_n}, a maximal element in $\bar{\Delta}_{\epsilon n}^{n-1}$. The colection $\{\Delta_{\epsilon n}^{n-1}\}$ may be ordered by inclusion. Consider $\epsilon_n \to 0$, and $\gamma_{\epsilon n} \in \bar{\Delta}_{\epsilon n}^{n-1} \subset \bar{\Delta}^{n-1}$, since $\bar{\Delta}^{n-1}$ is a compact set, there exists $\gamma \in \bar{\Delta}^{n-1} = \{\lambda \in R_+^n \sum_{i=1}^n \lambda_i = 1\}$ and a subnet $\{\gamma_{\epsilon_n'}\}$ such that $\gamma_{\epsilon_n'} \to \gamma$. If we prove that: $\gamma \in \Delta_{++}^{n-1} = \{\lambda \in \bar{\Delta}^{n-1}$ and $\lambda >> 0\}$ and that $e(\gamma) = 0$, then the theorem follows. Suppose that $\gamma \in \partial\bar{\Delta}^{n-1} = \{\lambda \in \bar{\Delta}^{n-1}$ and that least one $\lambda_i = 0 \, i \in \{1,\ldots,n\}\}$. It is strightforward from the definition that $\lim_{\lambda \to \partial\bar{\Delta}^{n-1}} \| e(\lambda) \| = \infty$ since e is bounded above, (see lemma 2), then there exists $\xi \in \Delta_{++}^{n-1}$ and ϵ_0 such that $\xi \; e\, (\gamma_{\epsilon''}) < 0$, $\forall\, \epsilon'' < \epsilon_0$. Since $\xi \in \bar{\Delta}_{\epsilon'''}^{n-1}$, $\forall\, \epsilon''' < \epsilon_0' \leq \epsilon_0$, the last inequality contradicts the maximality of $\gamma_{\epsilon''}$.

[1] See, for instance, Baiochi and Capelo (1989).

Suppose now there exists a $e_i(\gamma) < 0$ $i = \{1,\ldots, n_s\}$ then for the same $\xi \in \Delta_{++}^{n-1}$ we have that $\xi e(\gamma) > 0$. From the continuity of $\xi e(.)$ we obtain that $\xi e(\gamma_{\epsilon_0'}) < 0$, $\forall \epsilon_0' > \epsilon_0$, this contradicts the maximality of $(\gamma)_{\epsilon_0'}$. Then $e(\gamma) \geq 0$ follows. Since $\gamma \in S$ and $\gamma e(\gamma) = 0$, then $e(\gamma) = 0$.

The theorem is proved.

Then the set $E = \{\lambda : e(\lambda) = 0\}$ is non empty. That is, there exists at least one equilibrium, $(x(s, \lambda), p(s, \lambda))$, for ϵ..

6. UNIQUENESS FROM W.A.R.P.

Let us now to define the weak axiom of revealed preference (W.A.R.P.) from the excess utility function.

Definition 11. *We say that the excess utility function satisfies the W.A.R.P. if*

$$\lambda \,.\, e(\lambda_2) \geq 0 \ \ \textit{then} \ \ \lambda_2 \,.\, e(\lambda_1) < 0$$

Theorem 2. *W.A.R.P. implies uniqueness of equilibrium.*

Proof: We argue by contradiction. Suppose an λ_1 and λ_2 equilibria.

From Proposition 6) we have that $e(\lambda_1) = e(\lambda_2) = 0$.

Then $\lambda_i e(\lambda_j) = 0$, thus W.A.R.P. yields the following inequality $\lambda_j e(\lambda_i) < 0$, $i = \{1, 2\}$, $j = \{1, 2\}$.

Uniqueness follows.

Definition 12. Let e be an excess utility function, then e is monotone on $T_\lambda = \{\bar{\lambda} \in R^n : \bar{\lambda}\lambda = 0\}$ if $(\lambda_1 - \lambda_2)\,(e(\lambda_1) - e\,(\lambda_2)) > 0$, whenever $\lambda_1 - \lambda_2) \in T_\lambda$, $e(\lambda_1) \neq (\lambda_2)$.

Proposition 3 *If* $(e(.))$ *is a monotone function,* $e(\,.)$ *has W.A.R.P.*

Proof: Suppose that $\lambda_2 e(\lambda_1) \geq 0$. Since $\lambda_i \lambda > 0$; $i = 1, 2,$, there exists $\alpha > 0$ such that $\lambda_1 - \alpha\lambda_2 \in T_\lambda$. Hence, $(\lambda_1 - \alpha\lambda_2)\,(e(\lambda_1) - e(\alpha\lambda_2)) > 0$

follows, and then $-\lambda_1 e(\alpha \lambda_2) > \alpha \lambda_2 e(\lambda_1) \geq 0$. Since e is a homogeneous degree zero function, $\lambda_1 e(\lambda_2) < 0$. We have concluded our proof.

6.1. Some Aplications

Proposition 4 *If the central planner chooses λ using the rule $\succ$ and if the excess utility function has W.A.R.P., then the λ choosen by the central planner is an equilibrium.*

From W.A.R.P. we have that $\bar{\lambda} e(\lambda) < 0$. That is $\bar{\lambda} \succ \lambda$.

Economies with W.A.R.P. in the Excess Utility Function

Example 1. *Suppose an economy characterized by the following utility functions:*

$U_i(x) = x(s)^{\frac{1}{2}}$, *Let endowments be* $w_1(s) = a\, s$ *and* $w_2(s) = (1-a)s$, *with* $0 < a < 1$, $s \in (0,1)$. *Let μ denote the Lebesgue measure.*

The excess utility function is,

$$e(\lambda) = \left\{ \int \frac{1}{2} x_1^{-\frac{1}{2}} (x_1 - w_1)\, d\mu(s), \int \frac{1}{2} x_2^{-\frac{1}{2}} (x_2 - w_2)\, d\mu(s) \right\}$$

From the first order condition:

$$x_i(s) = \frac{\lambda_i^2}{\lambda_1^2 + \lambda_2^2}\, s$$

Substituing in the above equation we obtain that:

$$e(\bar{\lambda}) = 0, \text{ iff } \bar{\lambda} = \left\{ \frac{\sqrt{a}}{\sqrt{a} + \sqrt{1-a}}, \frac{\sqrt{1-a}}{\sqrt{a} + \sqrt{1-a}} \right\}.$$

It i easy to see that:

$\bar{\lambda} e(\lambda) < 0\ \forall\, \lambda$, *ie.* $\bar{\lambda} \succ \lambda$.

Example 2 *For economies with utilities* $U_i(x) = Lgx$, $i = (1, 2)$ *we obtain W.A.R.P. in the excess utility function.*

7. CONCLUDING REMARKS

In economies with infinite dimensional consumption spaces, the agent's budget may not be compact. Hence the existence of a demand function need not be a consequence of the utility maximization problem. In our approach without assuming its existence, with a simple proof, we have obtained the existence of the competitive equilibrium. So the excess utility function appears as a powerful tool in order to obtain a deeper insight into the structure of the equilibrium set. Some additional assumptions about the behavior of the excess utility function allow us to obtain a sufficient condition for uniqueness of the Walrasian equilibria. Unfortunately, its economic interpretations are not straightforward.

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